Workshop

Topological structures in mathematics, physics and biology

Rectangular diagrams of surfaces and mirror diagrams

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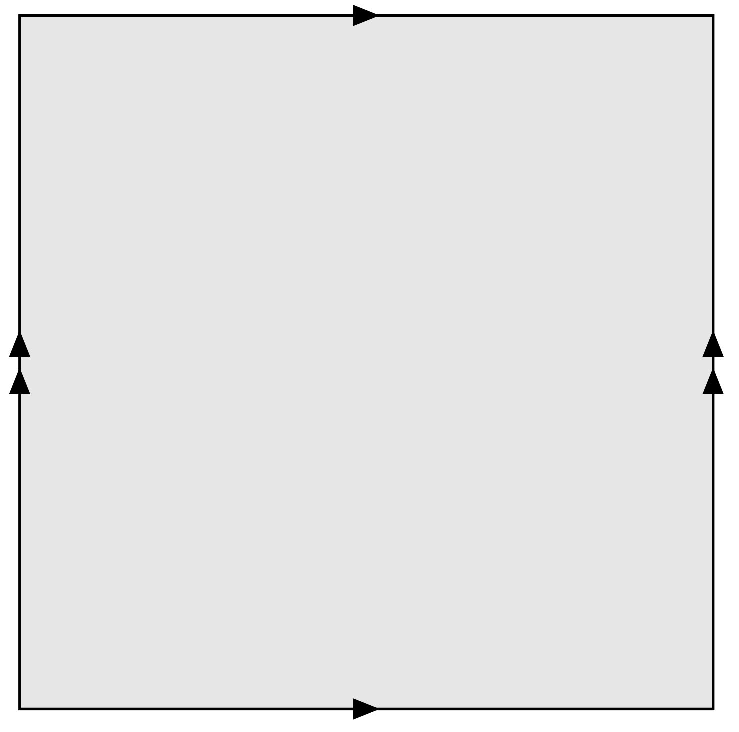
Steklov Mathematical Institute of Russian Academy of Sciences

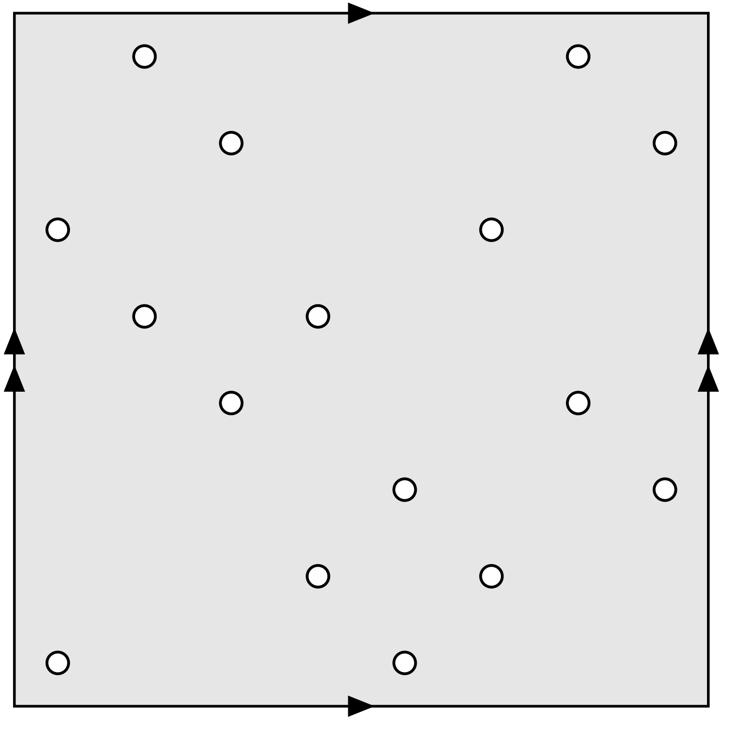
Berdsk, September 17, 2018

Collaborator: Maxim Prasolov (Moscow State University)

All diagrams discussed below live on the two-torus $\mathbb{T}^2 = \mathbb{S}^1 \times \mathbb{S}^1$:



Rectangular diagram of a link



$$\mathbb{S}^3 = \mathbb{S}^1 * \mathbb{S}^1 = \mathbb{S}^1 \times \mathbb{S}^1 \times [0,1]/\sim$$

$$(\theta, \varphi', 1) \sim (\theta, \varphi'', 1), \qquad (\theta', \varphi, 0) \sim (\theta'', \varphi, 0)$$

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The knot represented by a rectangular diagram R is $R \times [0,1]/\sim$:

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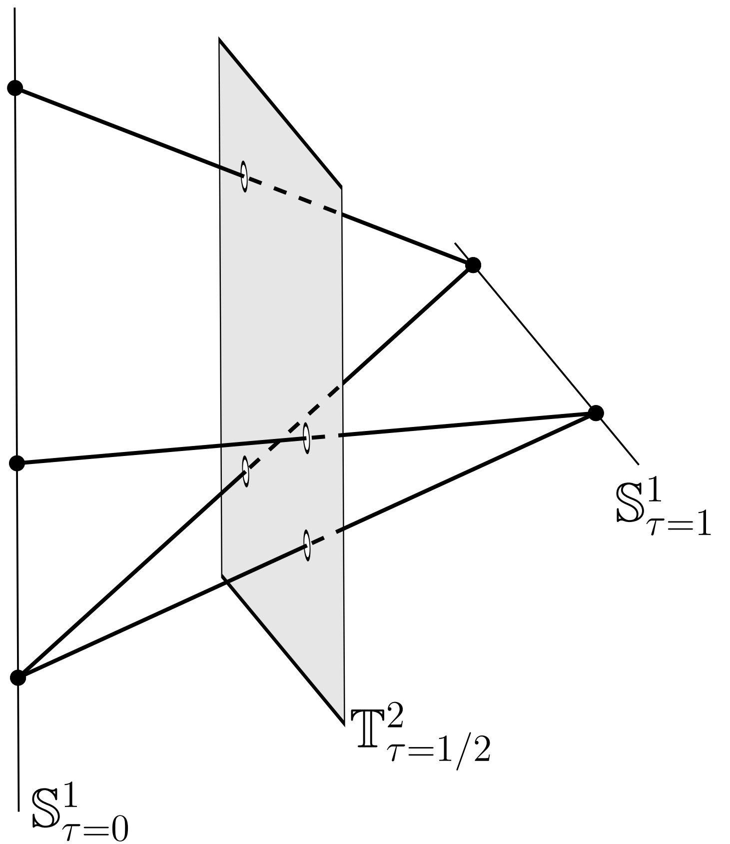
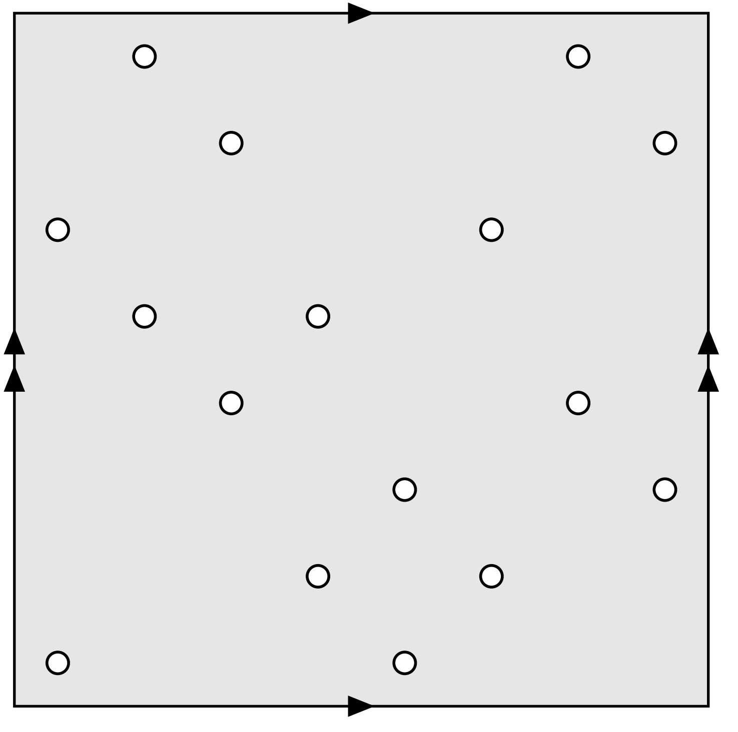


Converting a rectangular diagram into a conventional planar diagram



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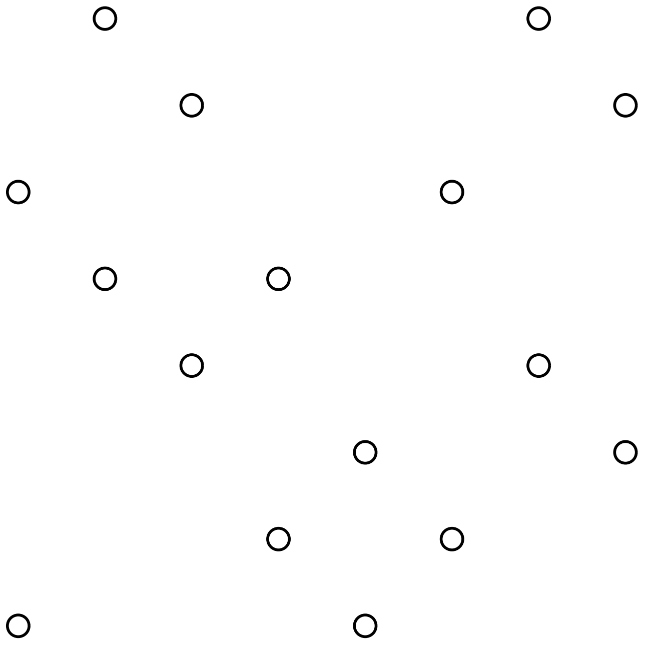

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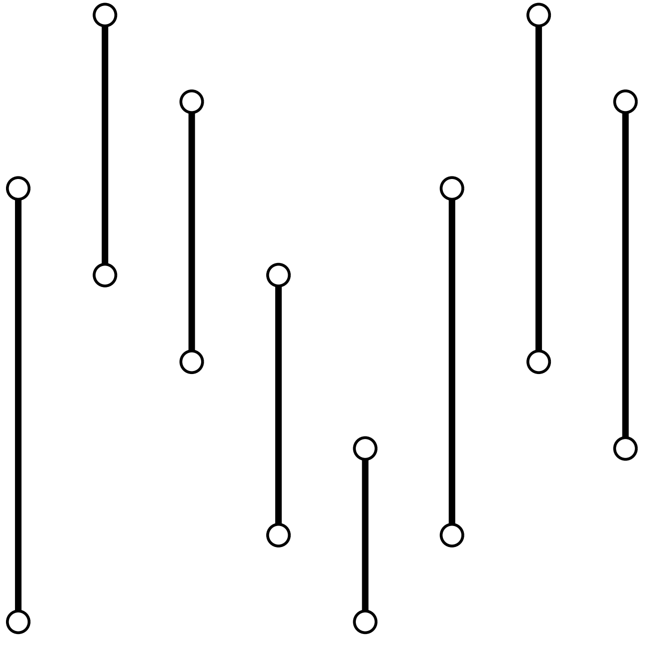

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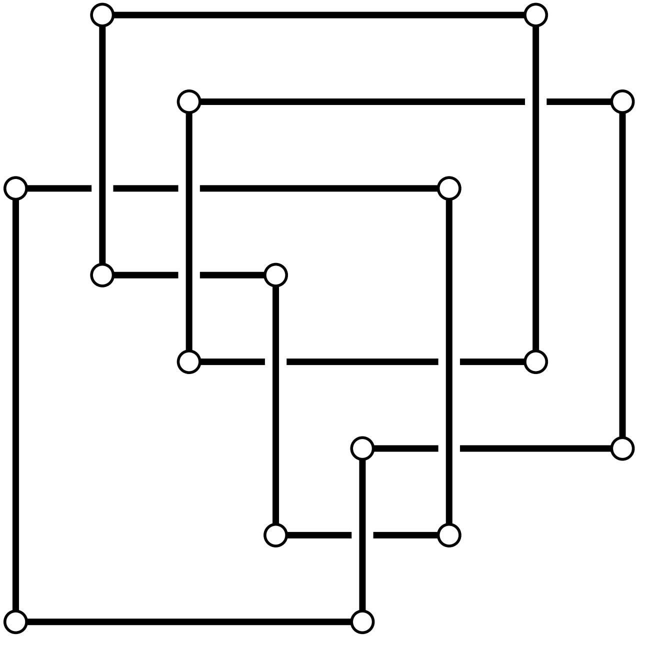
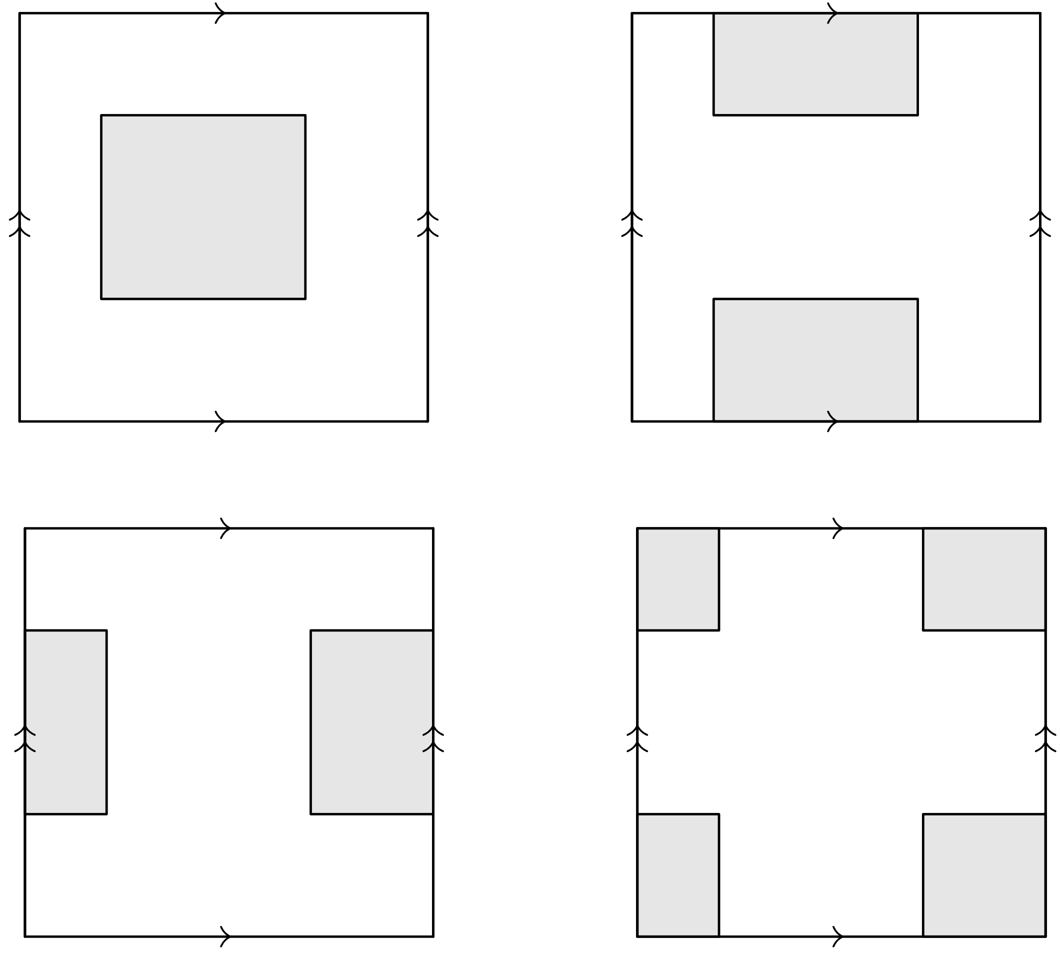
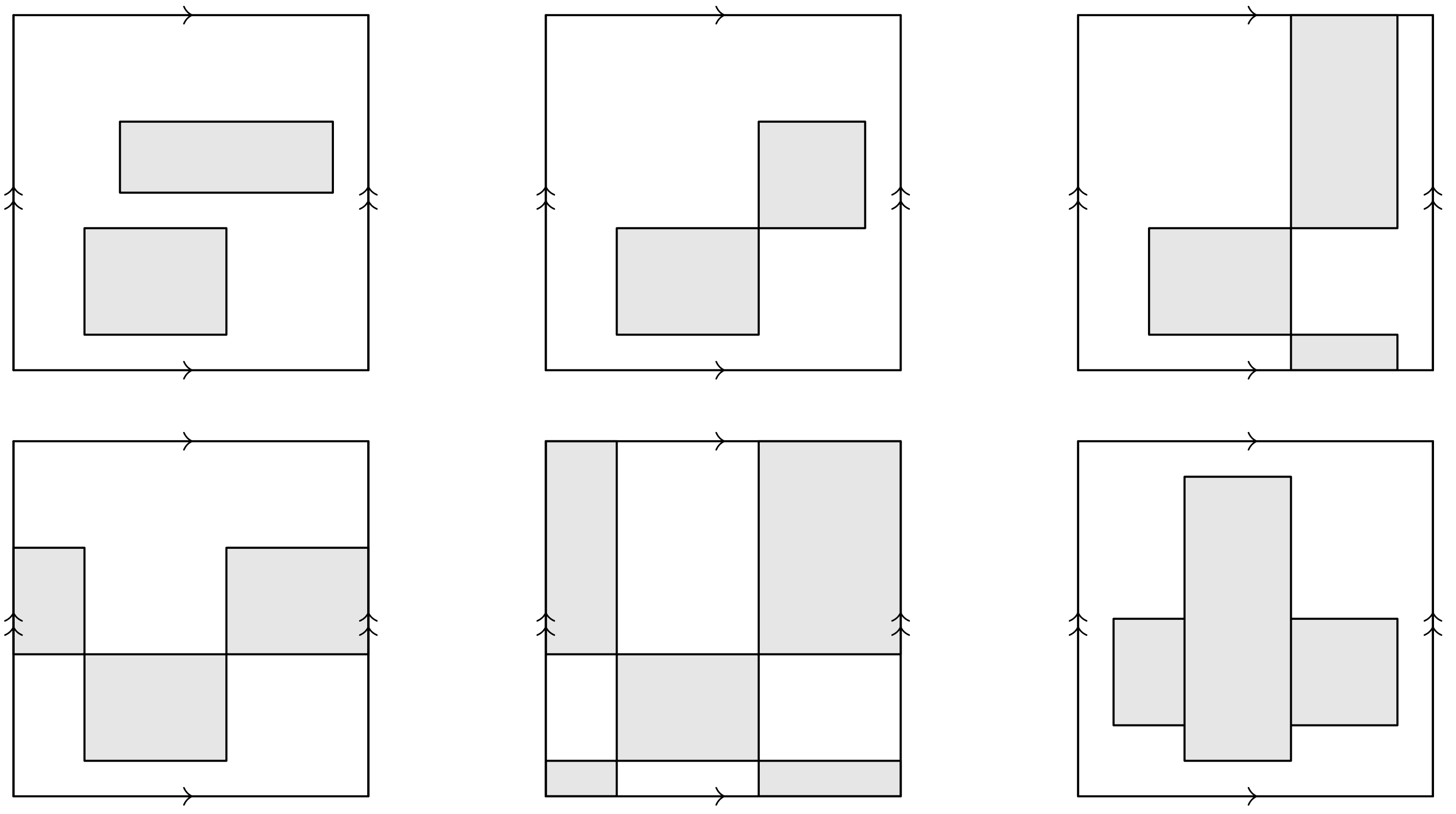


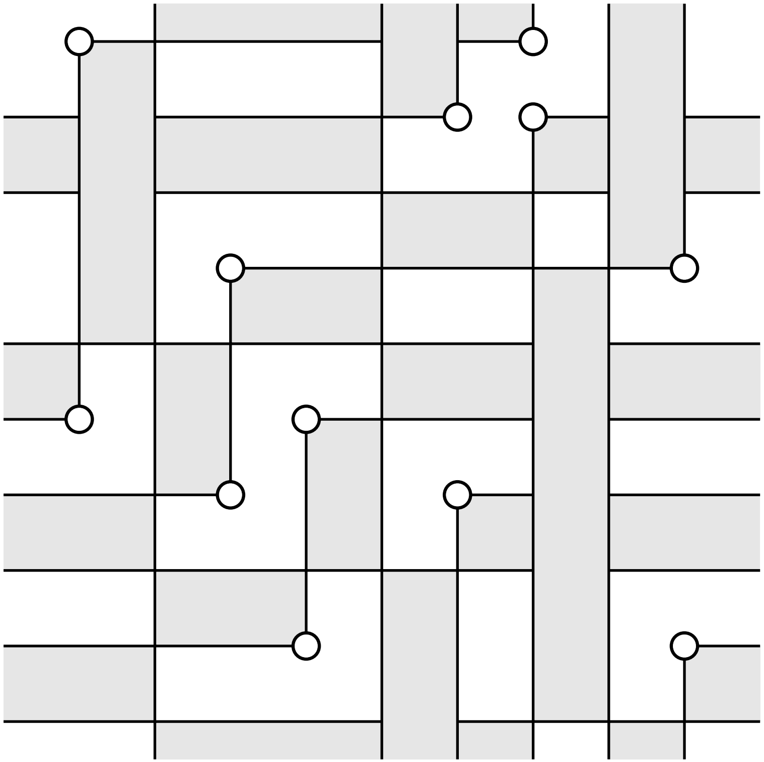
A rectangle in $\mathbb{T}^2$



Compatible rectangles



Rectangular diagram of a surface



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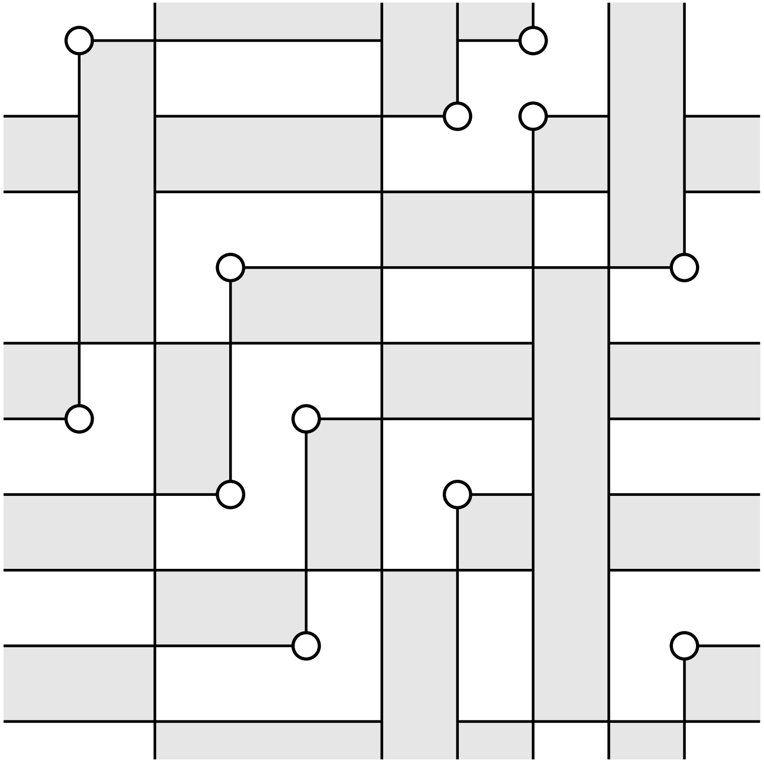
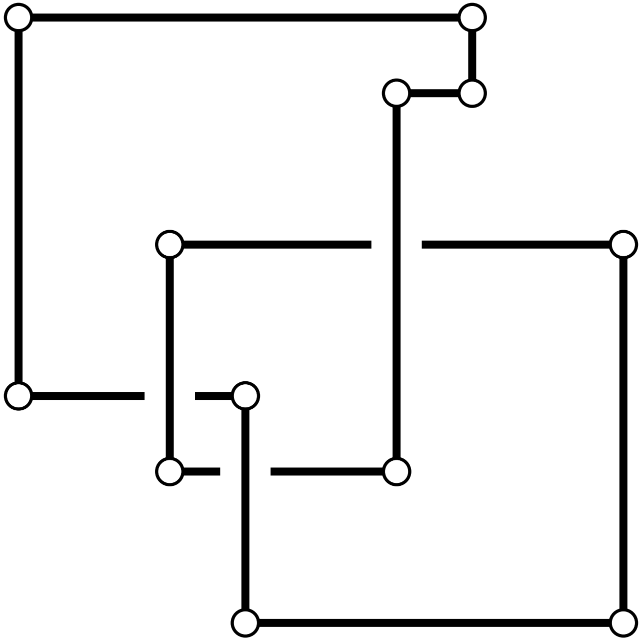
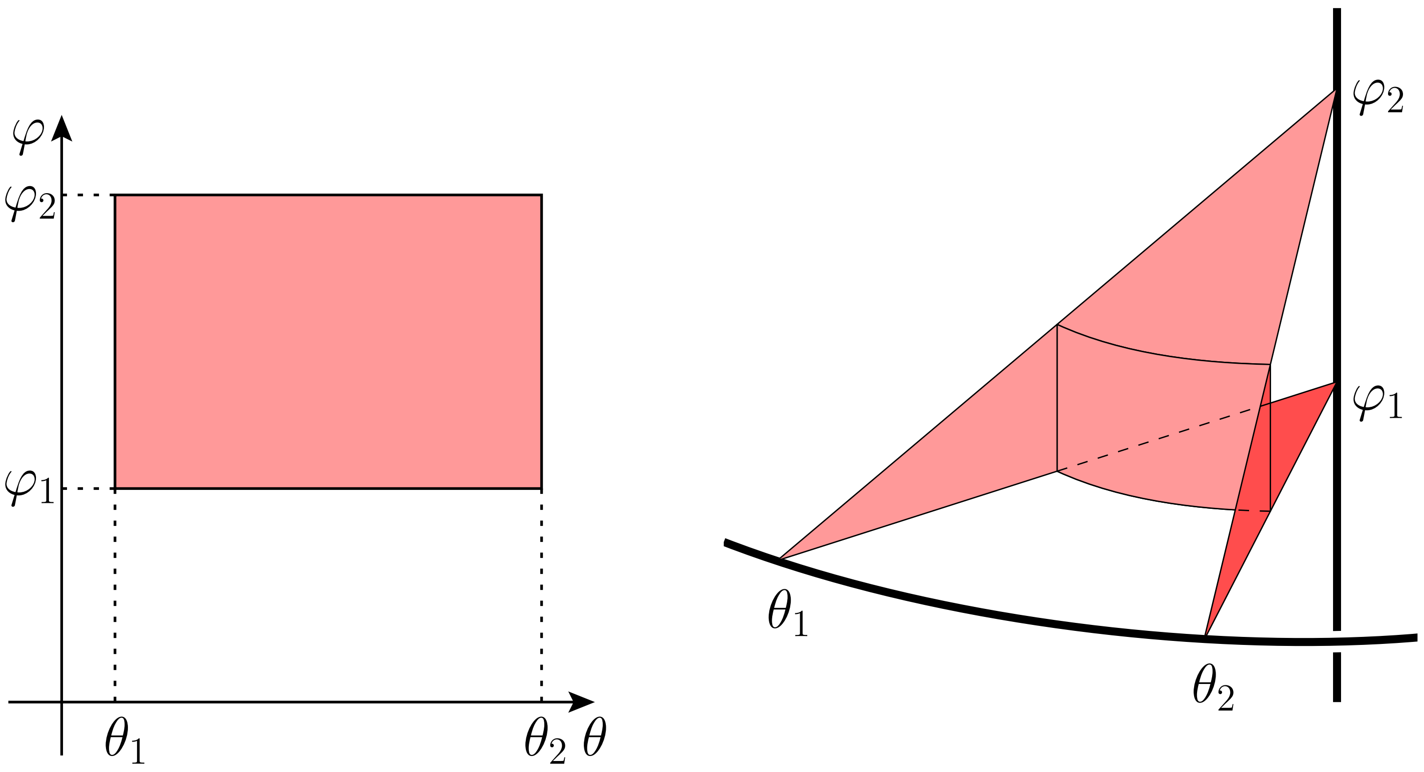




Constructing a surface from a rectangular diagram of a surface



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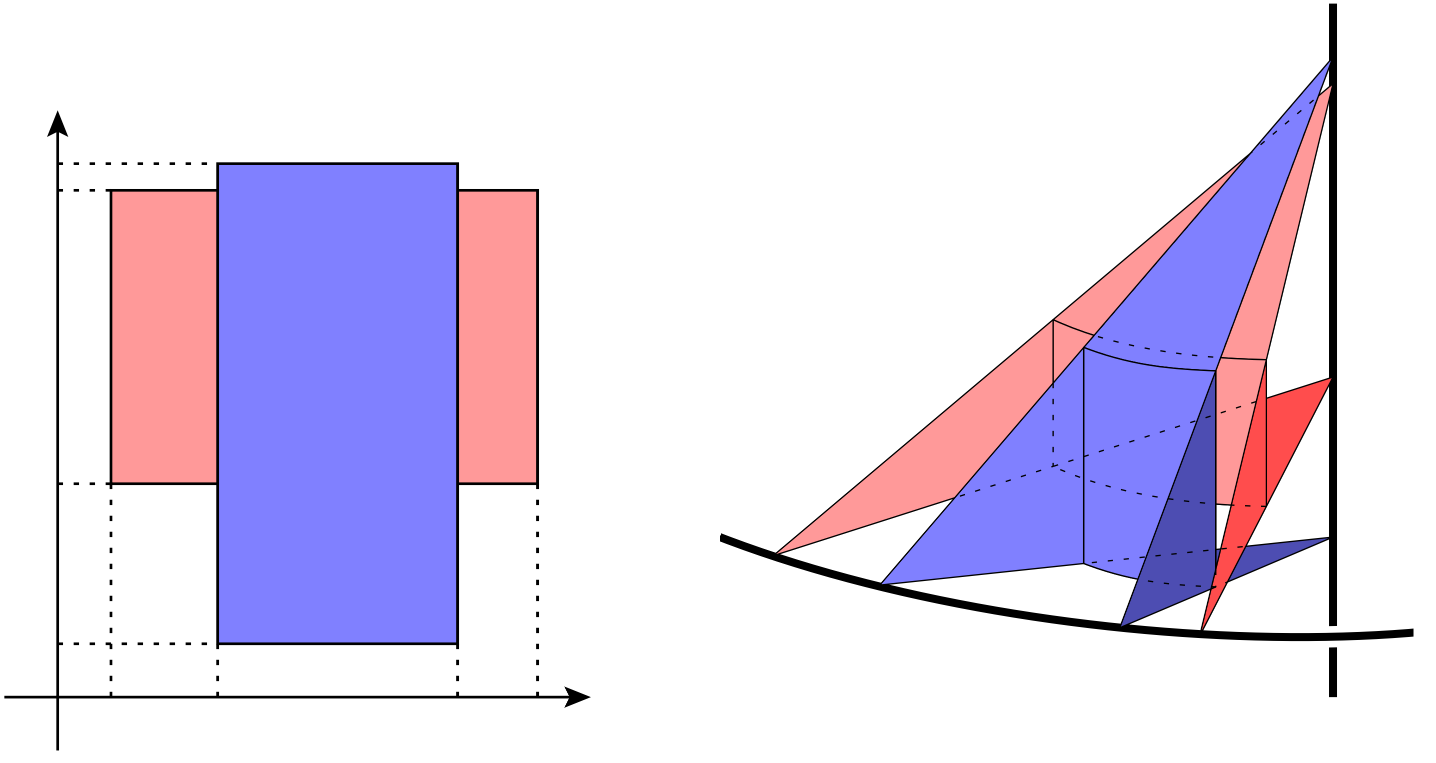
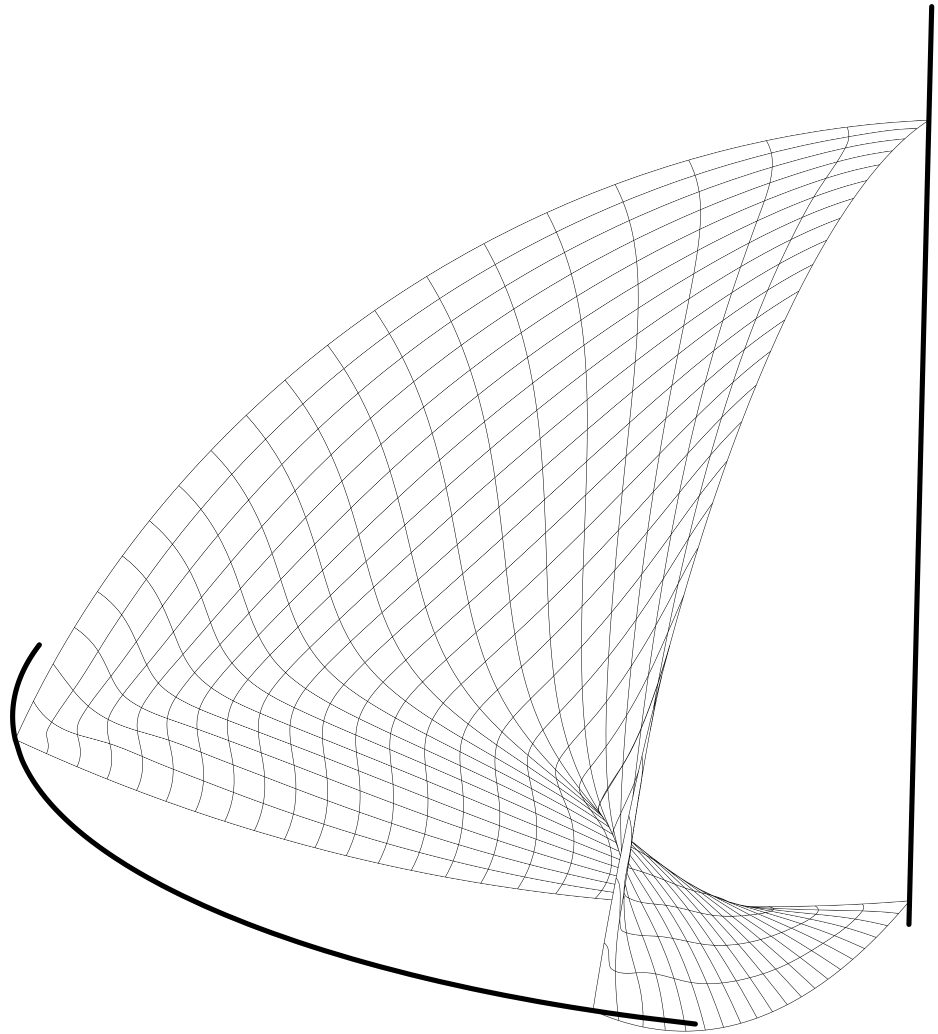
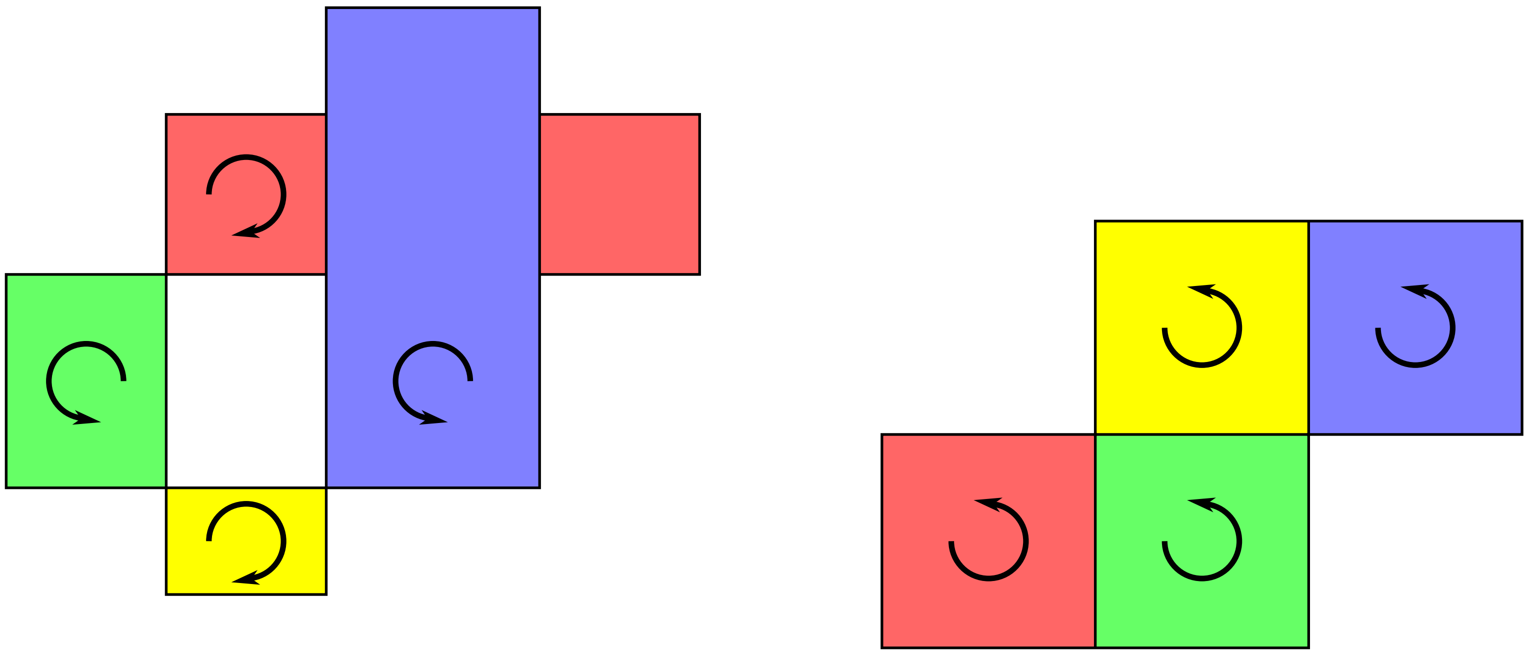


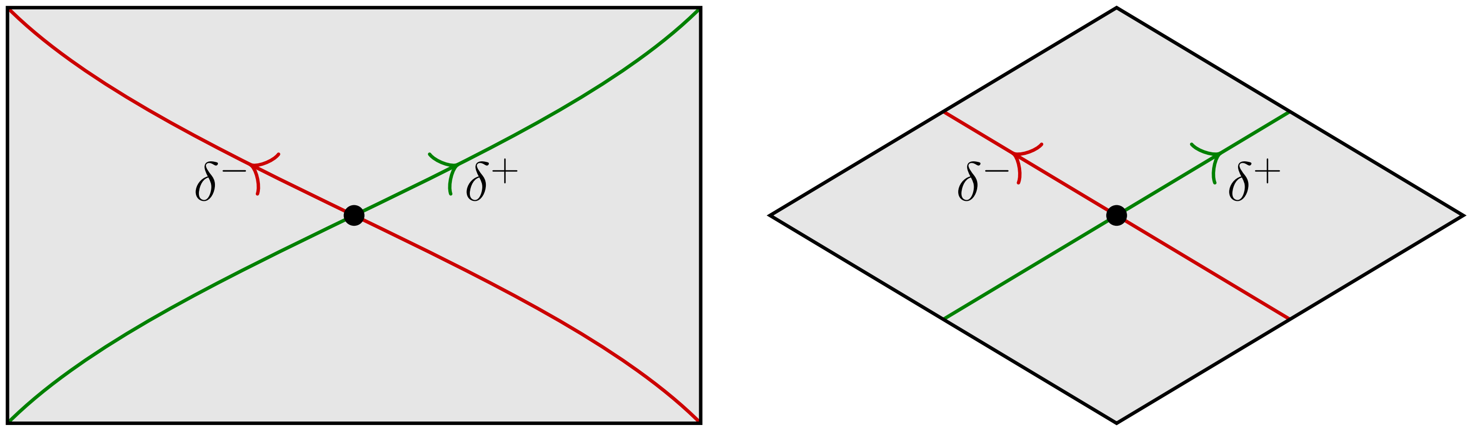
A smooth tile



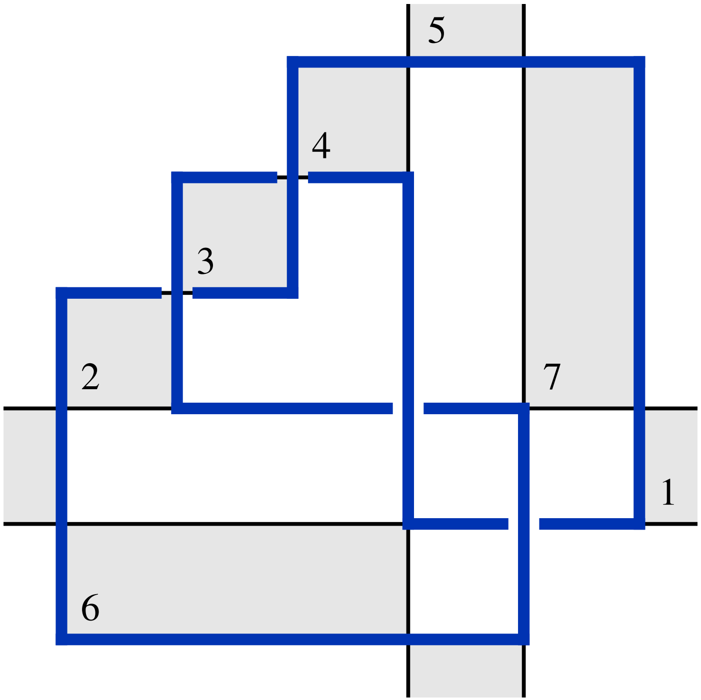
The surface comes with a tiling

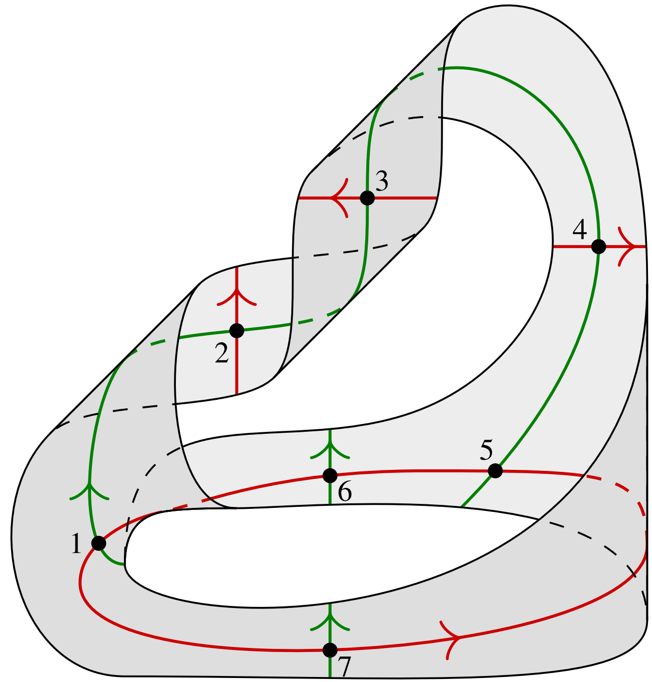


Canonic dividing configuration



Example





Theorem.
Compact surfaces in $\mathbb{S}^3$ / isotopy =
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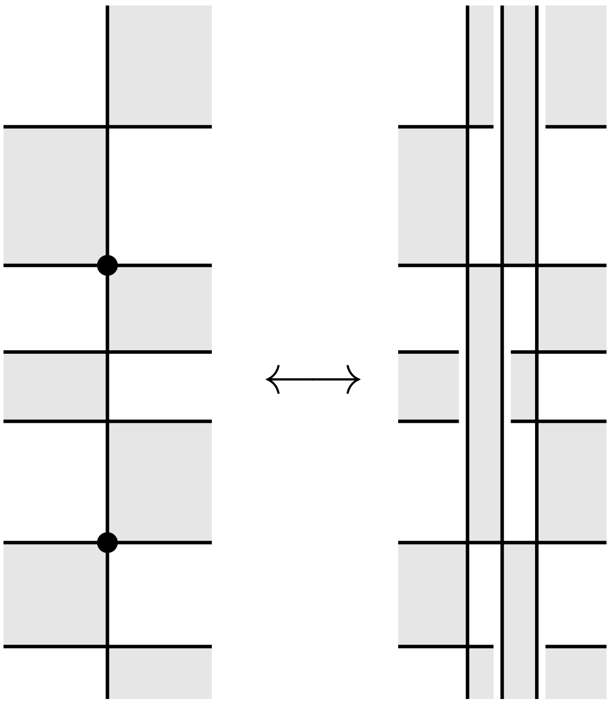
Basic moves include:

- (half-)wrinkle creation and wrinkle reduction moves;
- stabilizations and destabilizations;
- exchange moves;
- flypes.

Wrinkle creation and wrinkle reduction moves



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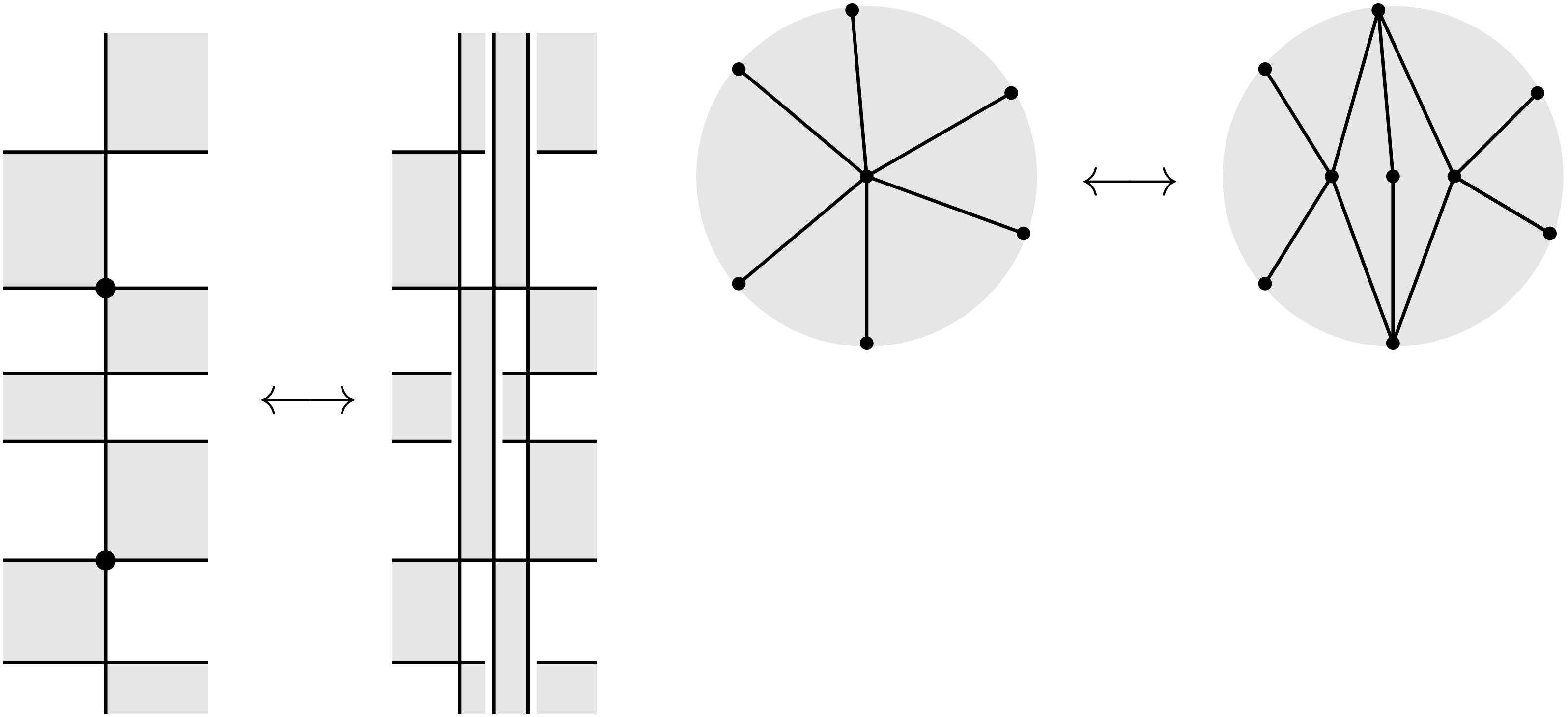

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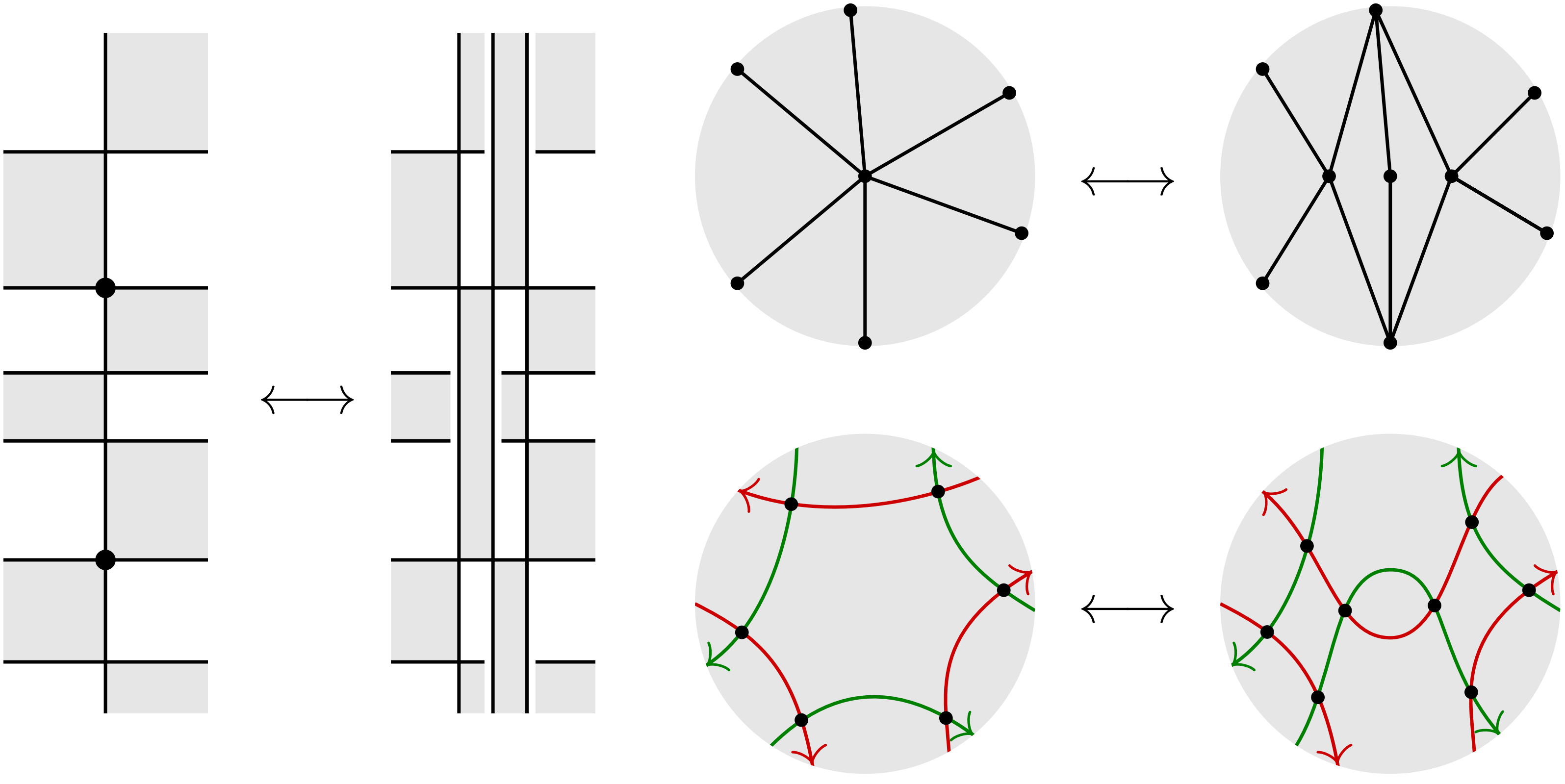
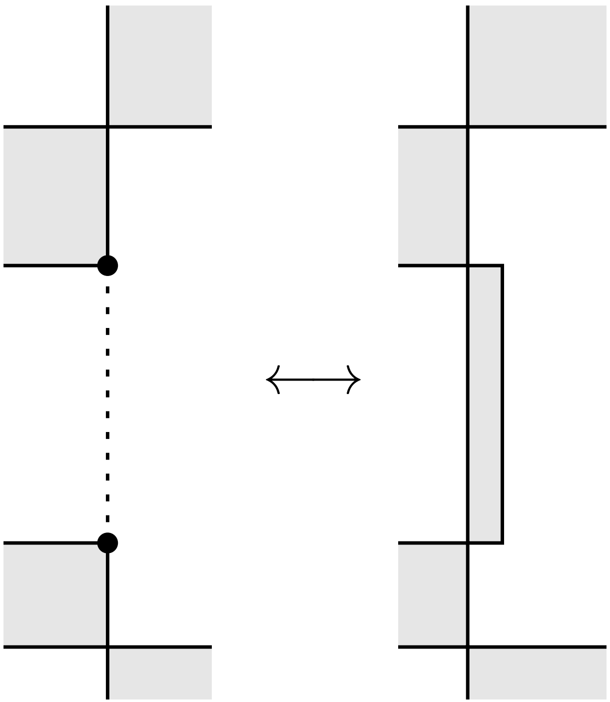


Half-wrinkle creation and half-wrinkle reduction moves



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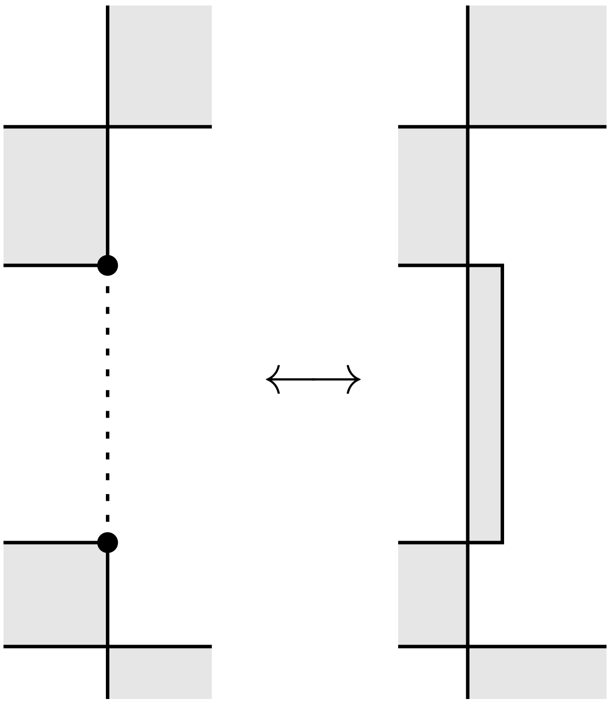

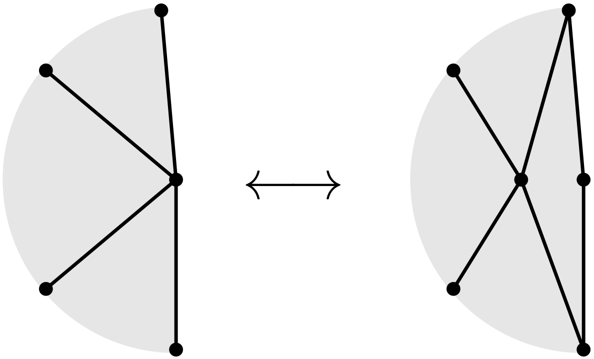

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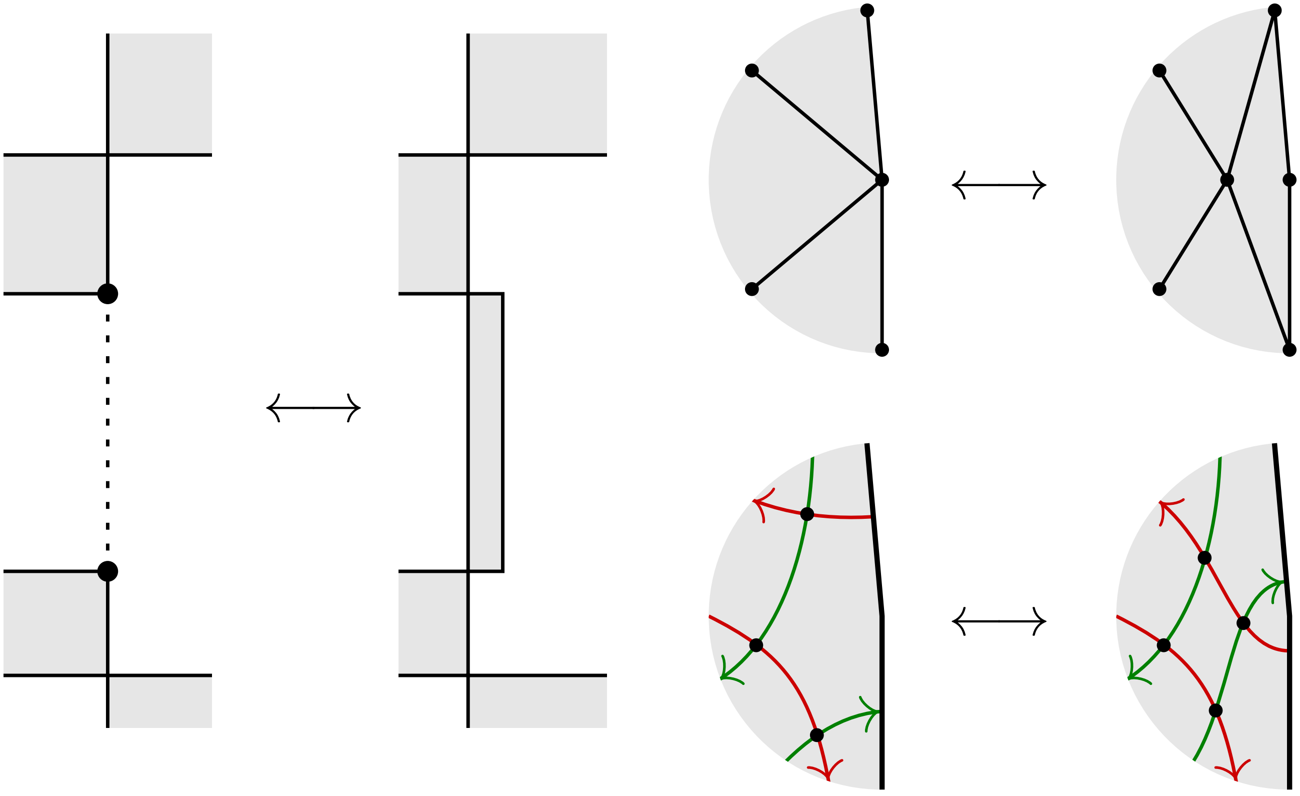
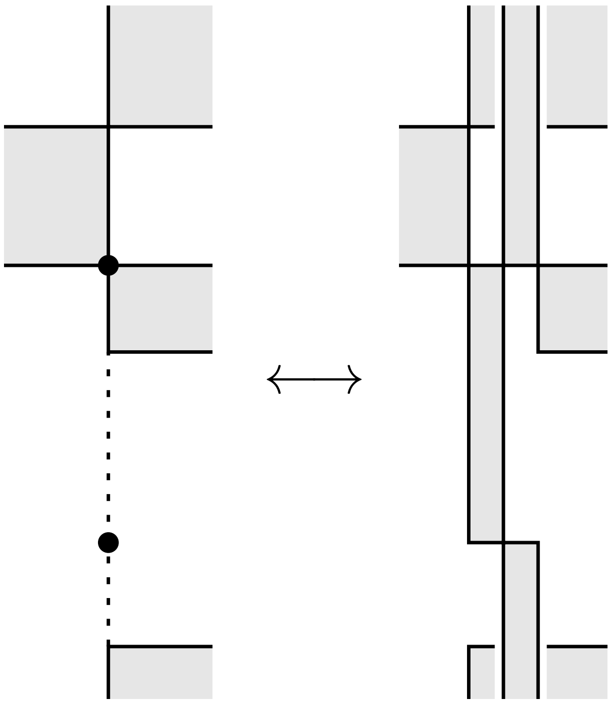


Stabilization and destabilization moves



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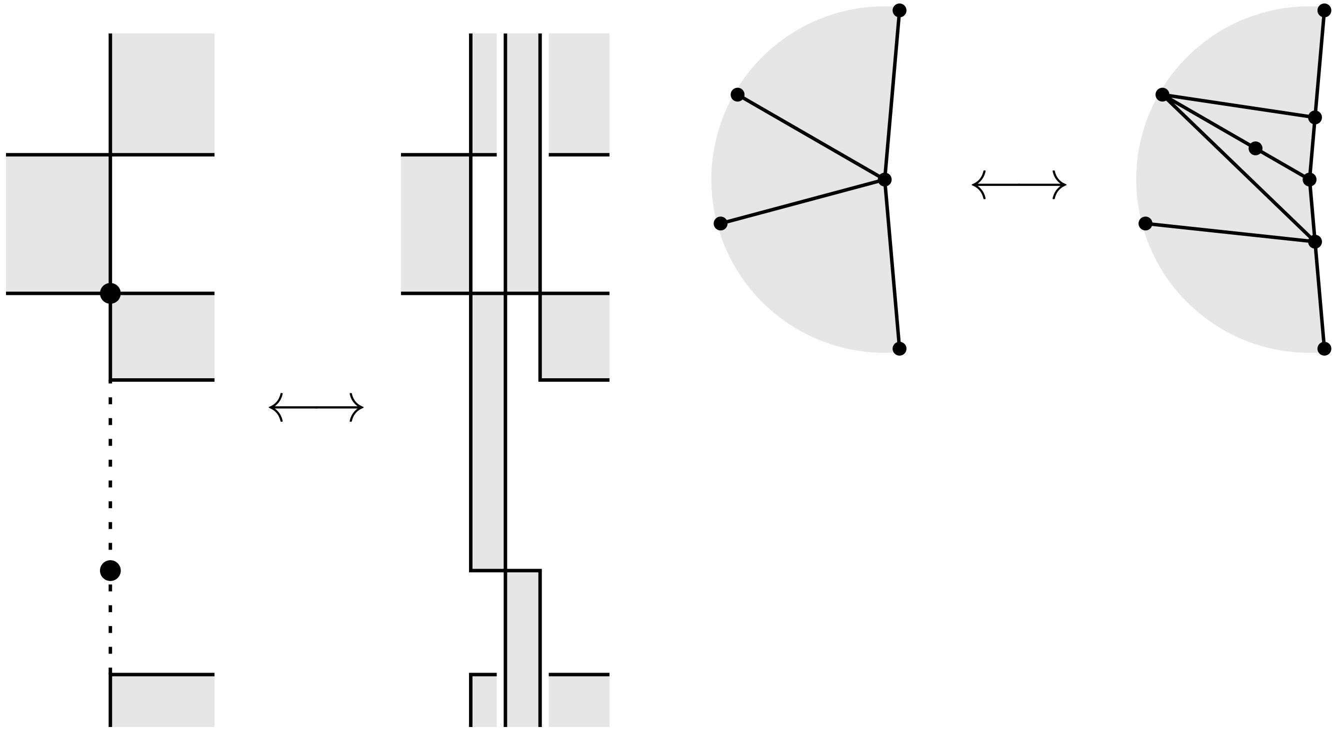

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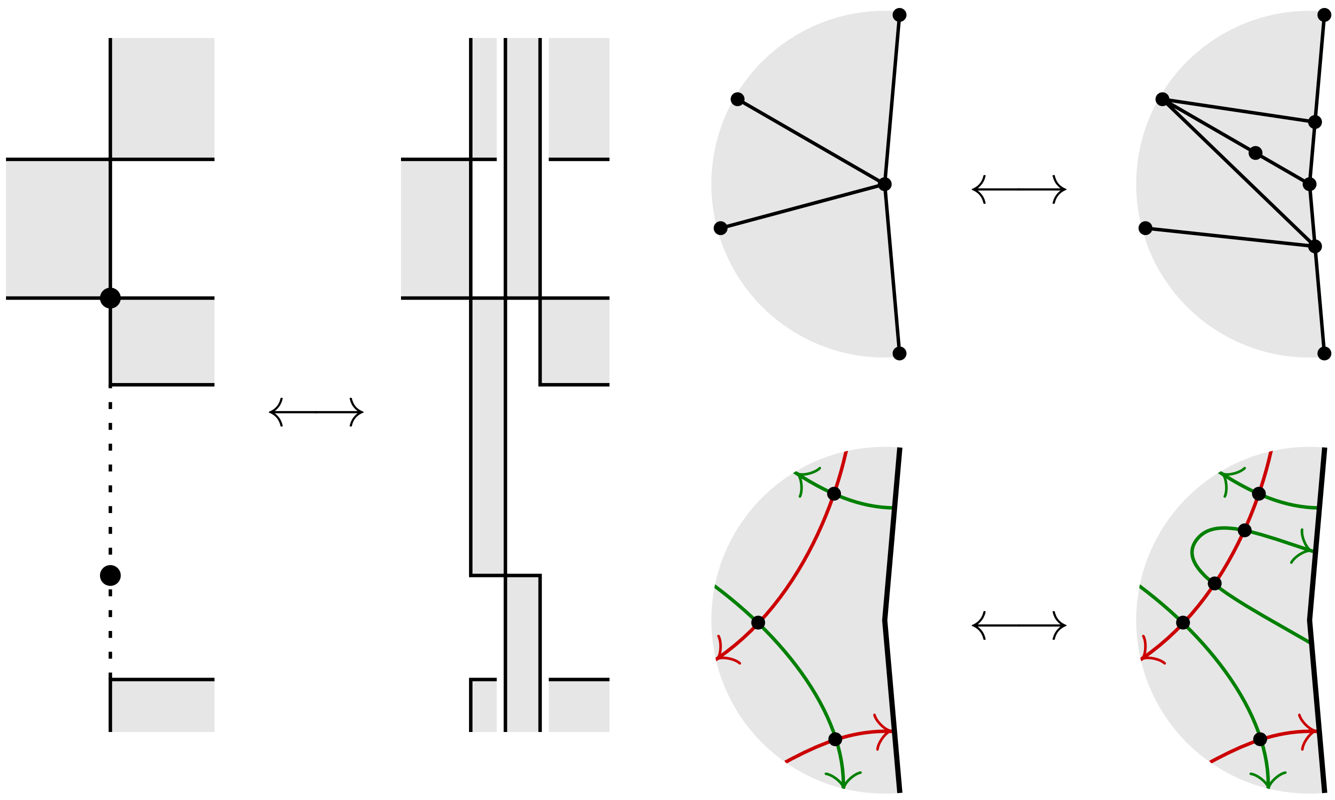
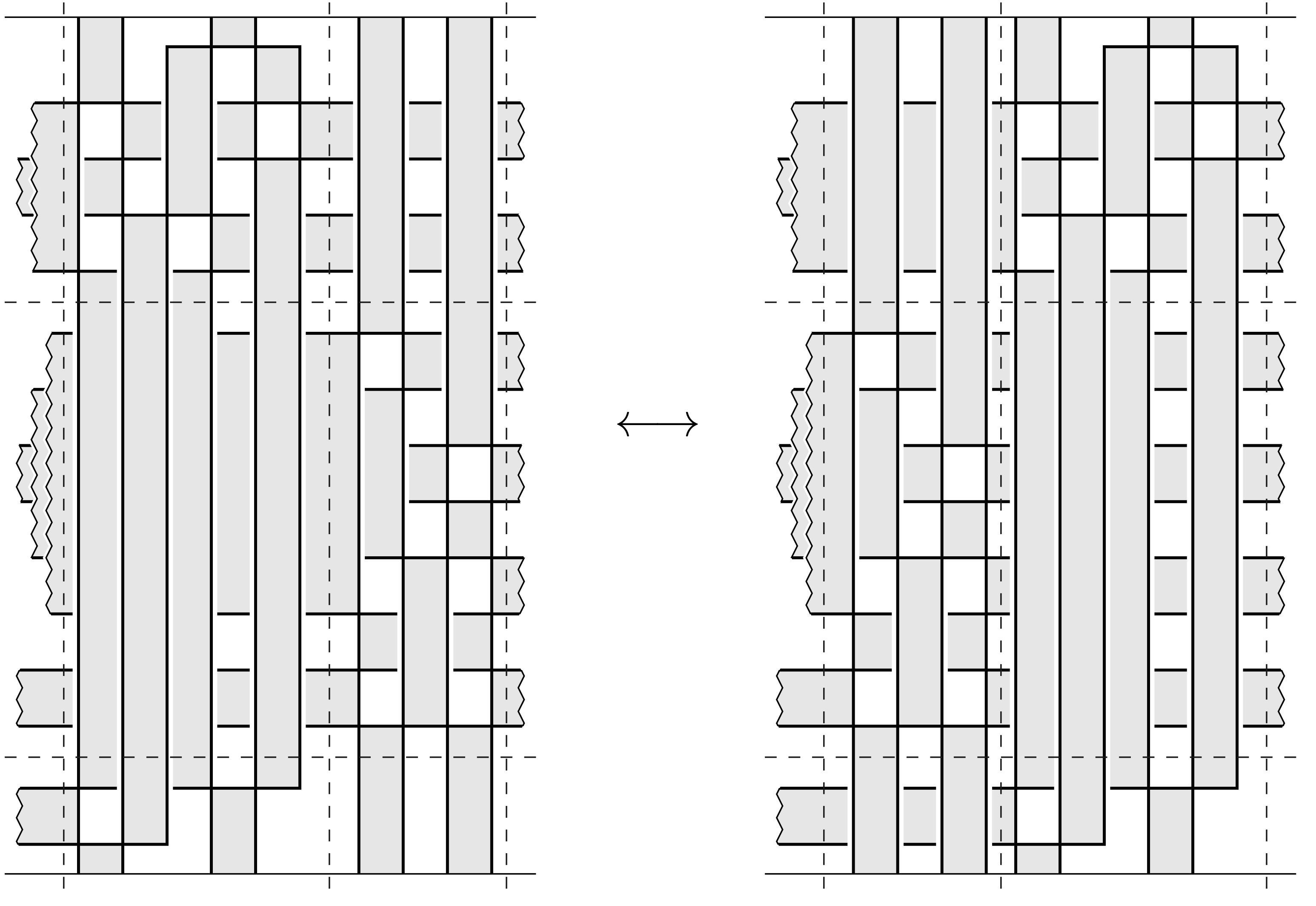
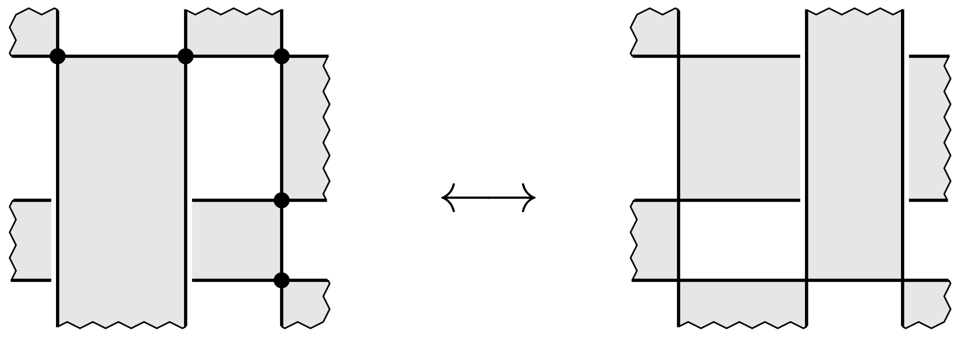


Exchange moves



Flypes



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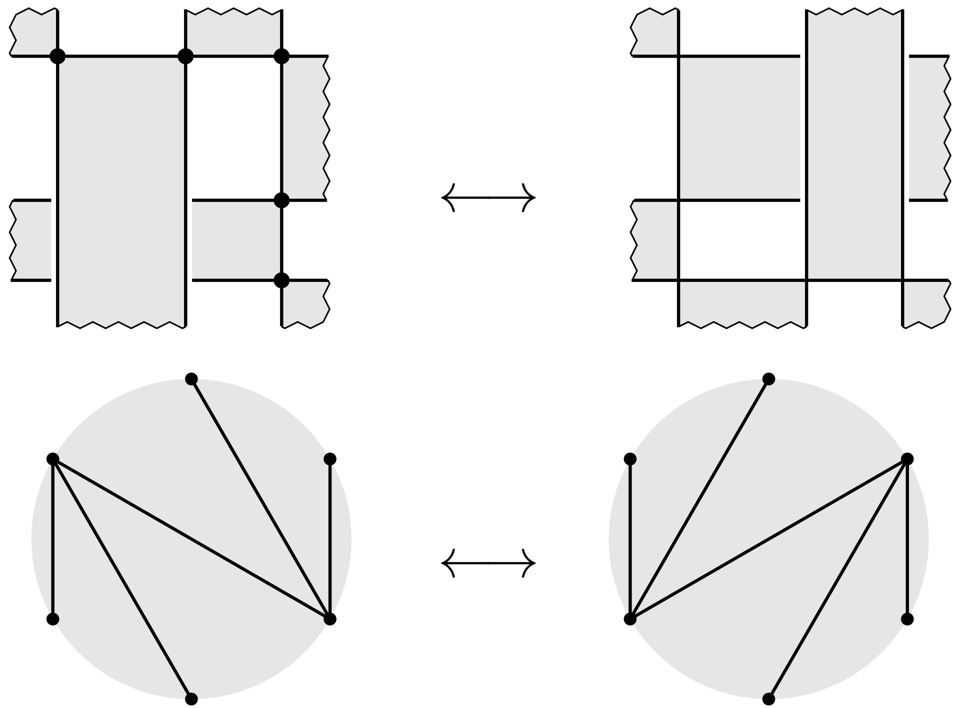

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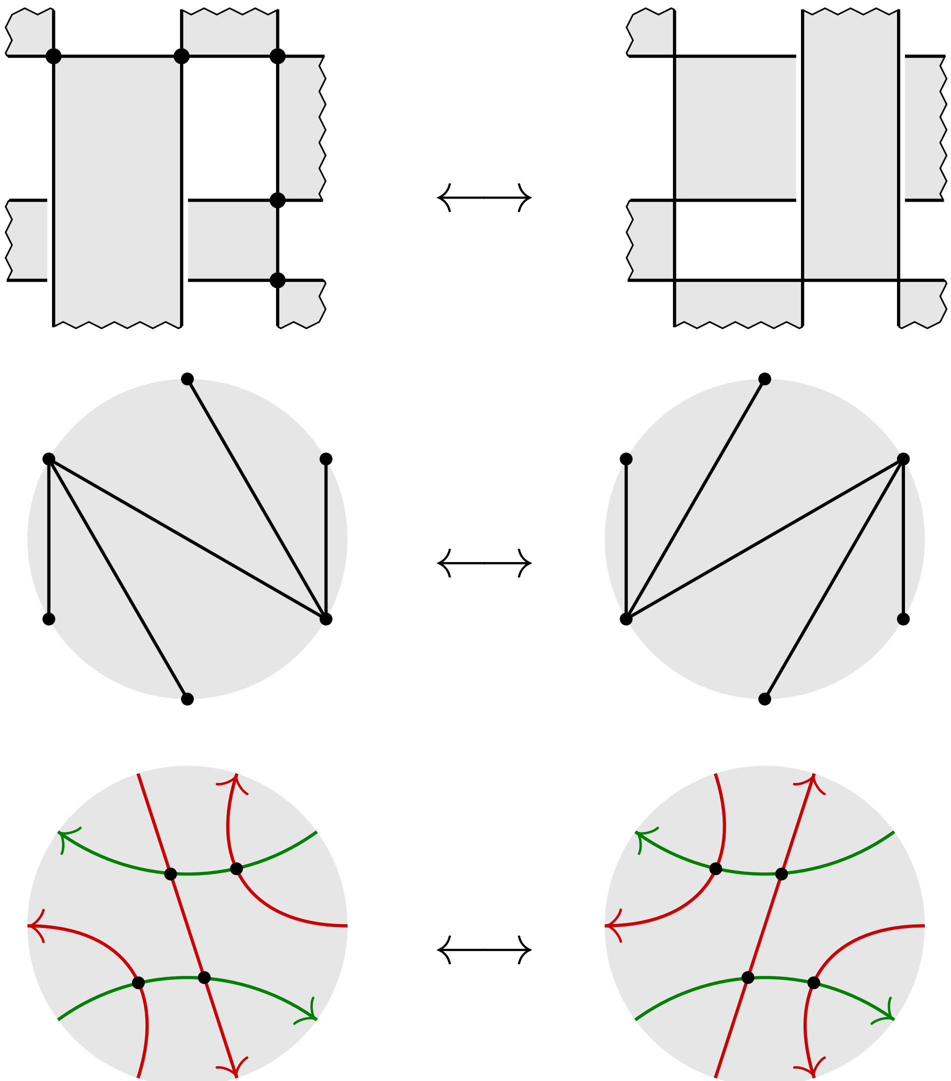
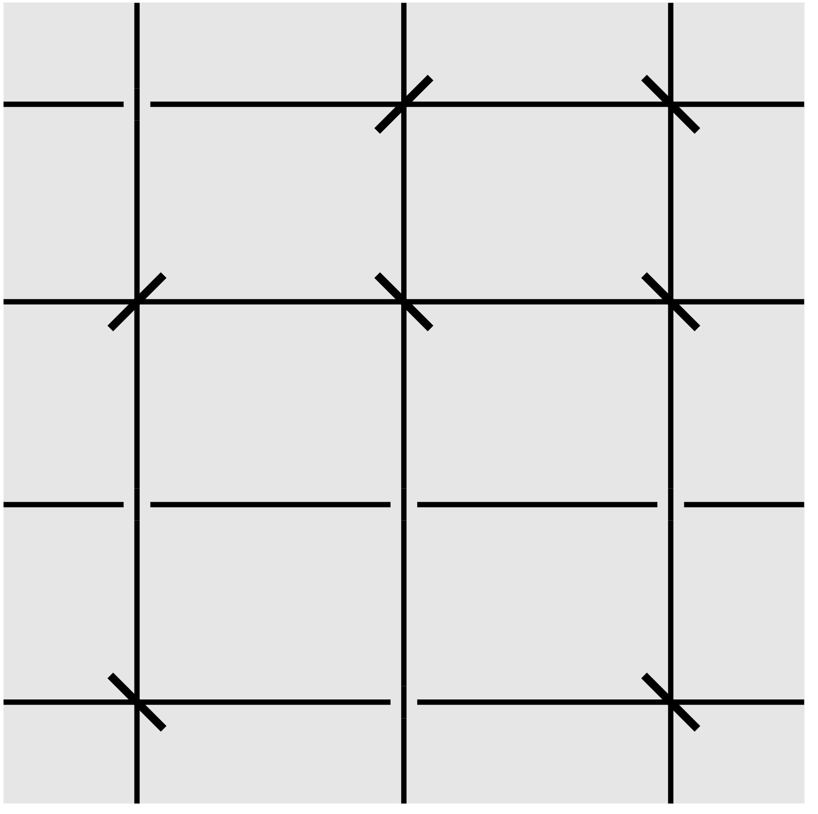
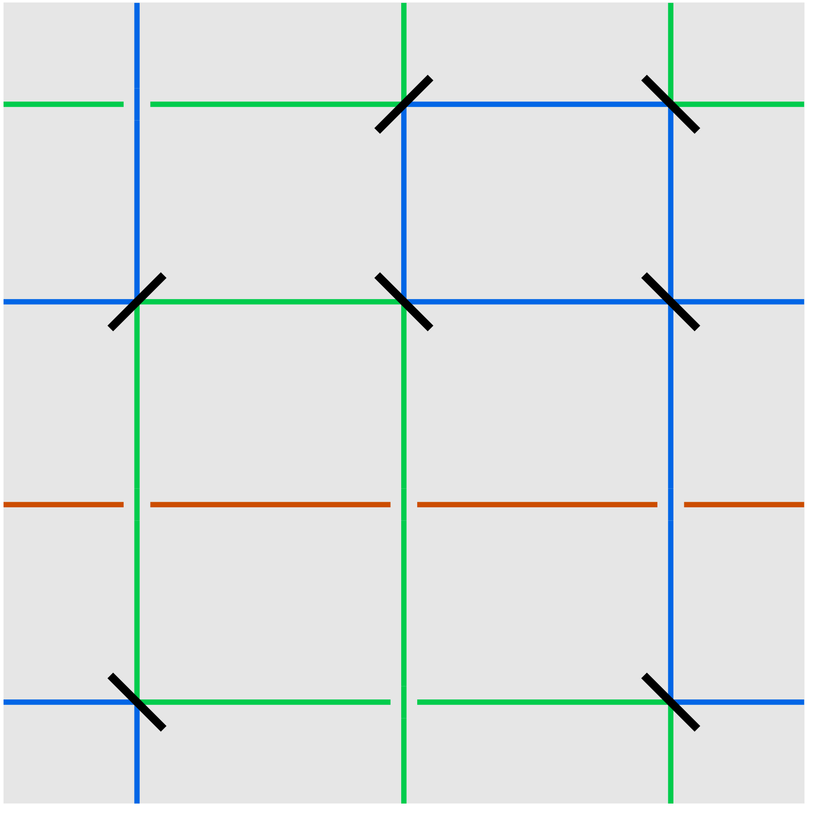


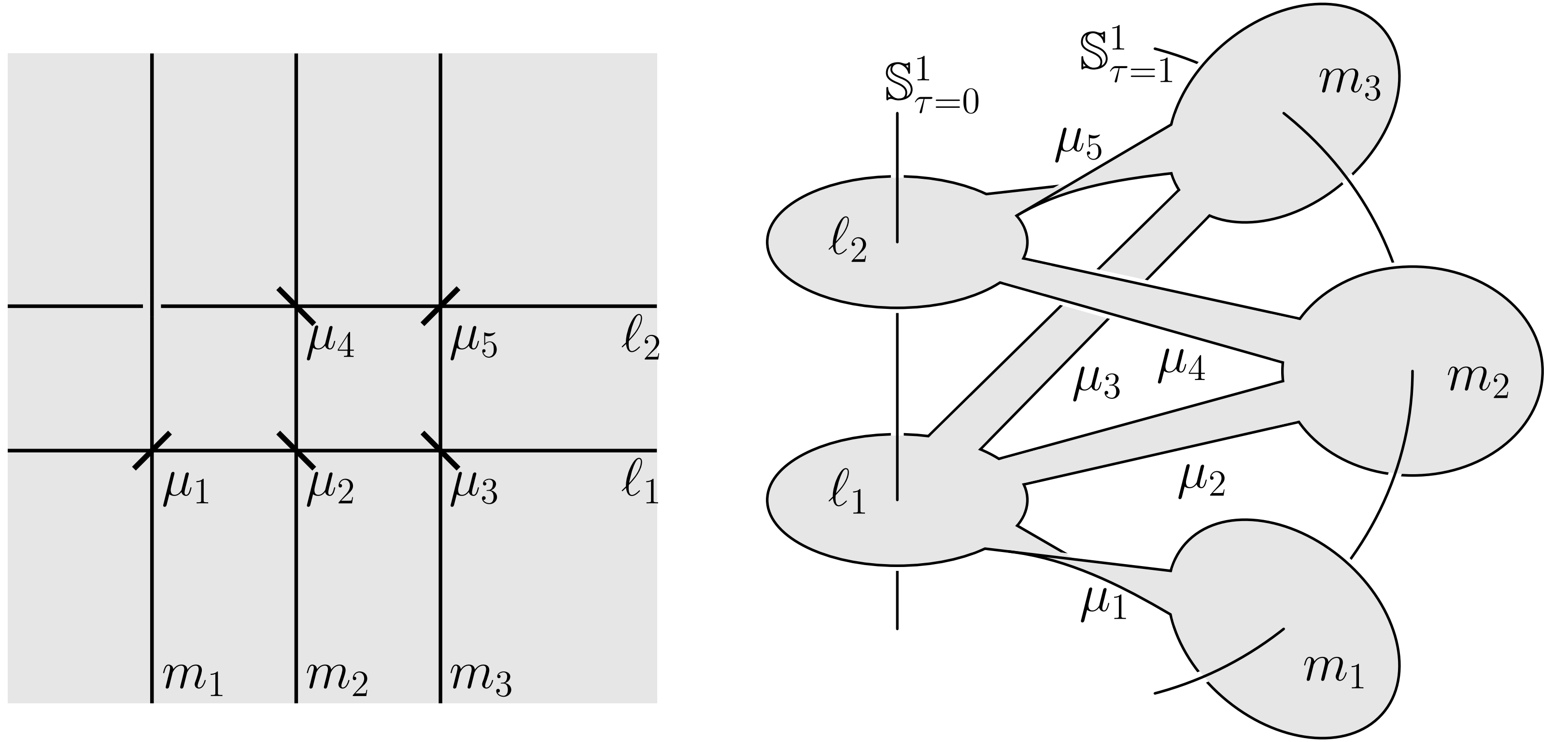
Mirror diagrams



Boundary circuits



Mirror diagrams represent spatial ribbon graphs



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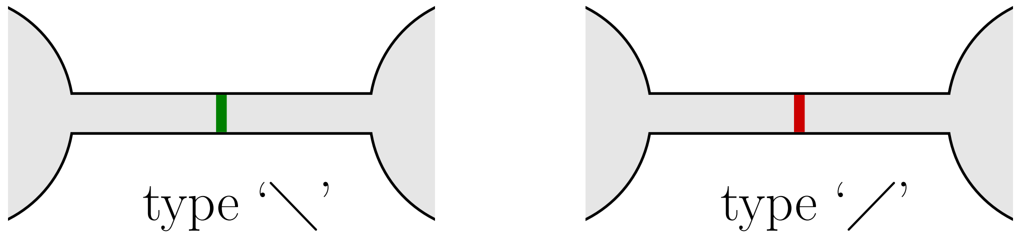

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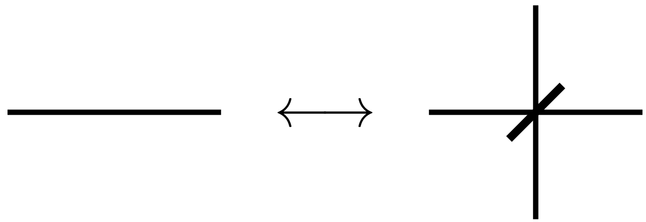
Elementary moves include:

- extension and elimination moves;
- elementary bypass addition/removal moves;
- slide moves.

Extension and elimination moves



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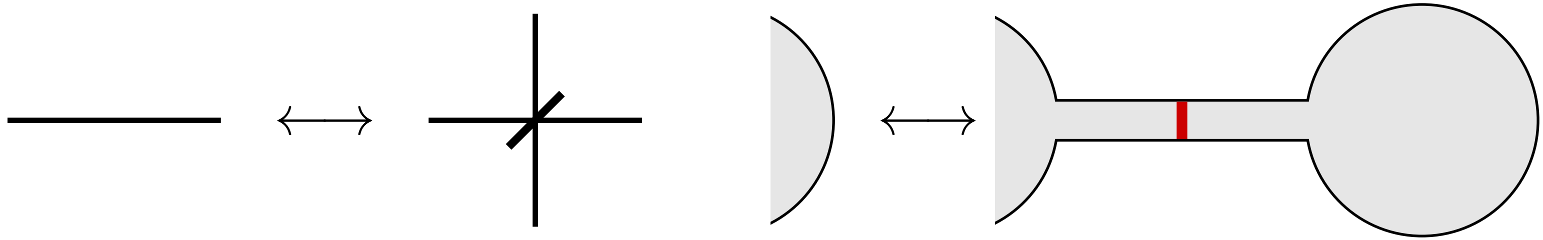
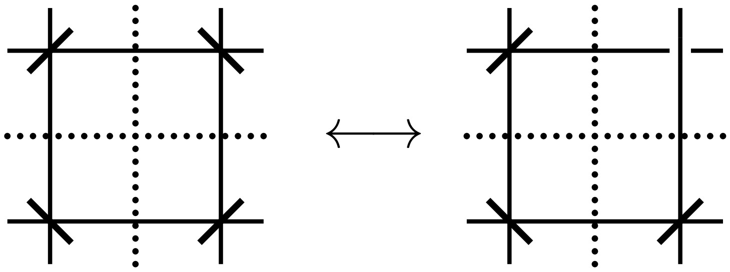


Elementary bypass addition/removal moves



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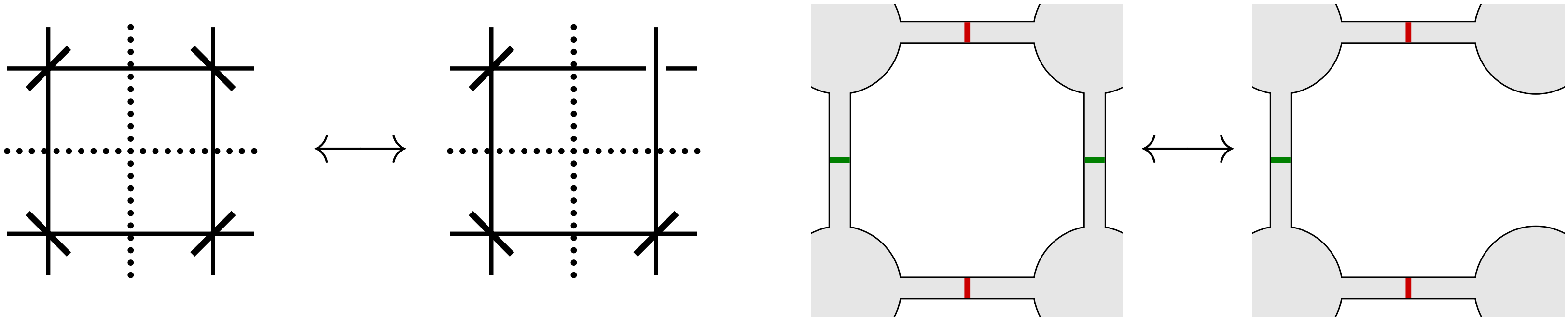
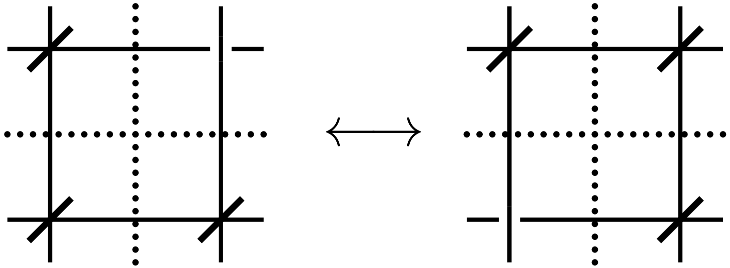


Slide moves



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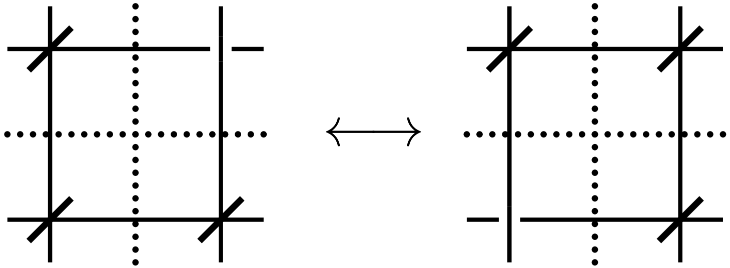

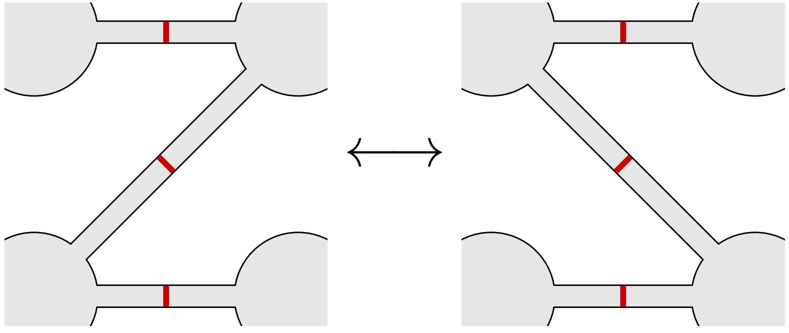

Types of moves

Type I moves: preserve (isotopy class of) δ^+, change δ^-

Type II moves: preserve (isotopy class of) δ^-, change δ^+

Neutral moves: preserve both δ^+ and δ^-

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Type I moves 'commute' with type II moves.

Giroux's convex surfaces

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Theorem.
Let $M^3 = \mathbb{S}^3$ and ξ be the standard contact structure (right-invariant 2-plane field on $\mathbb{S}^3 \cong SU(2)$). Then:
Giroux's convex surfaces with Legendrian boundary / convex isotopy =
rectangular diagrams of surfaces / neutral and type I moves.