Topological structures in mathematics, physics and biology

Topological surgery in cosmic phenomena

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  - Wormholes creation
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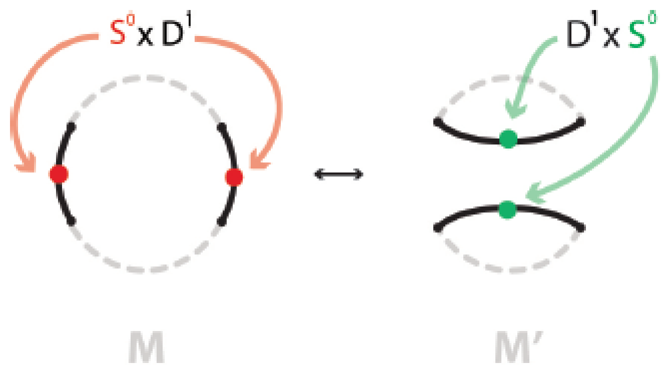
Definitions - *The formal definition of surgery*

Definition 1. An **m-dimensional n-surgery** is the topological process of creating a new m-manifold M' out of a given m-manifold M by removing a framed n-embedding $h : S^n \times D^{m-n} \hookrightarrow M$, and replacing it with $D^{n+1} \times S^{m-n-1}$, using the 'gluing' homeomorphism h along the common boundary $S^n \times S^{m-n-1}$. Namely, and denoting surgery by χ:

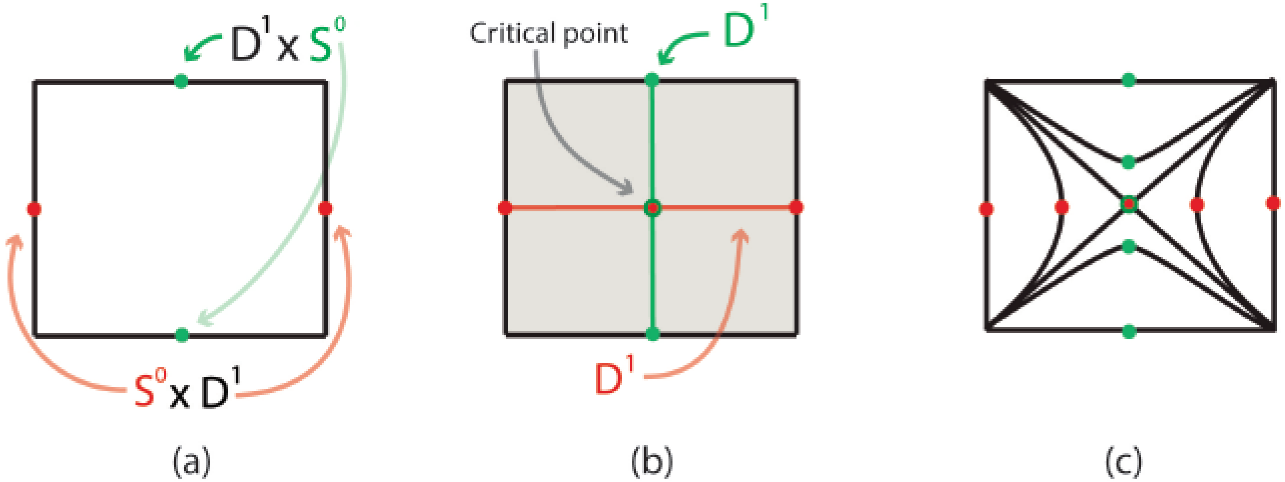
$$M' = \chi(M) = \overline{M \setminus h(S^n \times D^{m-n})} \cup_{h|_{S^n \times S^{m-n-1}}} (D^{n+1} \times S^{m-n-1}).$$



1-dimensional 0-surgery

Definitions - *The local process*

The process of surgery is the continuous passage within $D^{n+1} \times D^{m-n}$ from of boundary component $(S^n \times D^{m-n}) \subset \partial(D^{n+1} \times D^{m-n})$ to its complement $(D^{n+1} \times S^{m-n-1}) \subset \partial(D^{n+1} \times D^{m-n})$. More precisely, the boundary segments $(S^n \times D^{m-n})$ approach each other within the handle $D^{n+1} \times D^{m-n}$ until they touch at the critical point $D^{n+1} \cap D^{m-n}$ from which the complement boundary segment $(D^{n+1} \times S^{m-n-1})$ emerge.



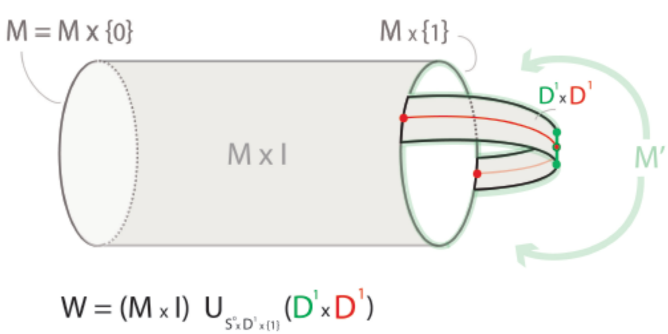
$$\partial(D^1 \times D^1) = S^1 = (S^0 \times D^1) \cup (D^1 \times S^0)$$

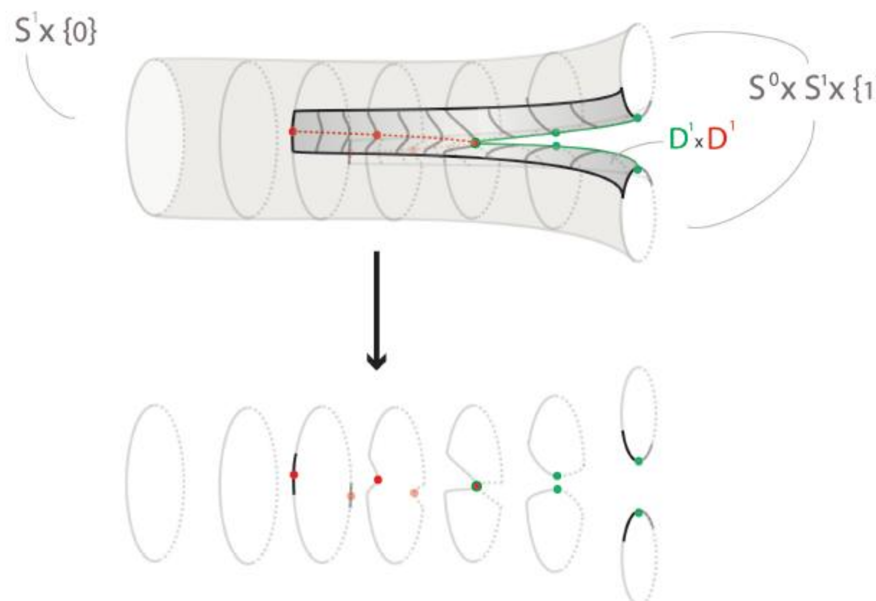
The local process of 1-dimensional 0-surgery

Definitions – *Local process as part of the global process*

Definition . An $(m+1)$-dimensional **cobordism** $(W; M, M')$ is an $(m+1)$-dimensional manifold W^{m+1} with boundary the disjoint union of the closed m-manifolds M, M': $\partial W = M \sqcup M'$.

Definition . The **trace** of the surgery removing $S^n \times D^{m-n} \subset M^m$ is the cobordism $(W; M, M')$ obtained by attaching the $(m+1)$-dimensional $(n+1)$-handle $D^{n+1} \times D^{m-n}$ to $M \times I$ at $S^n \times D^{m-n} \times \{1\} \subset M \times \{1\}$.





The cobordism $(W; S^1, S^0 \times S^1)$ and the process of 1-dimensional 0-surgery

Morse theory - *Local form of a Morse function*

Proposition . Let $f : W^{m+1} \to I$ be a Morse function on an $(m+1)$-dimensional cobordism $(W; M, M')$ between manifolds M and M' with

$$f^{-1}(\{0\}) = M, \quad f^{-1}(\{1\}) = M'$$

and such that all critical points of f are in the interior of W.

(i) If f has no critical points then $(W; M, M')$ is a trivial h-cobordism, with a diffeomorphism

$$(W; M, M') \cong M \times (I; \{0\}, \{1\})$$

which is the identity on M.

(ii) If f has a single critical point of index i then W is obtained from $M \times I$ by attaching an i-handle using an embedding $S^{i-1} \times D^{m-i+1} \hookrightarrow M \times \{1\}$, and $(W; M, M')$ is an elementary cobordism of index i with a diffeomorphism

$$(W; M, M') \cong (M \times I \cup D^i \times D^{m-i+1}; M \times \{0\}, M').$$

Morse theory – *Local form of a Morse function*

Theorem . Every m-dimensional manifold M^m admits a Morse function $f : M \to \mathbb{R}$.

Lemma 1. For any $0 \leq i \leq m+1$ the Morse function

$$f : D^{m+1} \to \mathbb{R}; (x_1, x_2, ..., x_{m+1}) \mapsto -\sum_{j=1}^{i} x_j^2 + \sum_{j=i+1}^{m+1} x_j^2$$

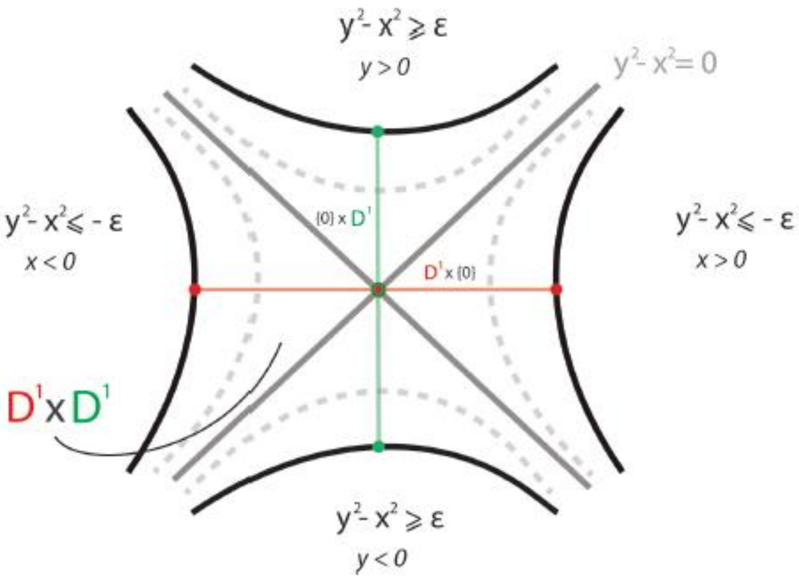
has a unique interior point $0 \in D^{m+1}$, which is of index i. The $(m+1)$-dimensional manifolds with boundary defined for $0 < \epsilon < 1$ by

$$W_{-\epsilon} = f^{-1}(-\infty, -\epsilon], W_\epsilon = f^{-1}(-\infty, \epsilon]$$

are such that W_ϵ is obtained from $W_{-\epsilon}$ by attaching an i-handle

$$W_\epsilon = W_{-\epsilon} \cup D^i \times D^{m-i+1}$$

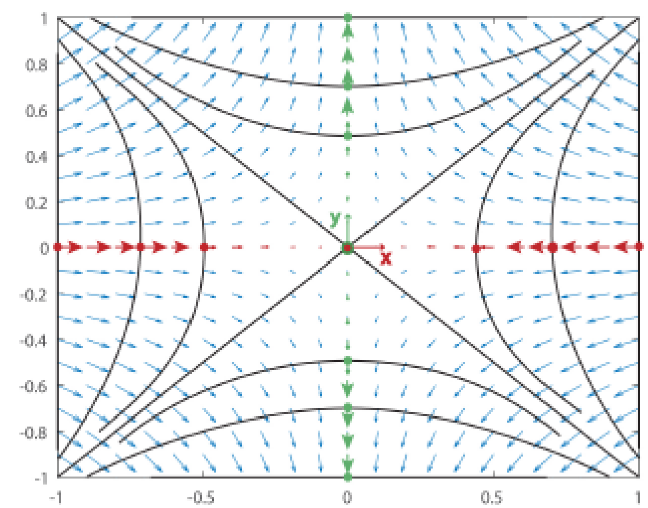
1-dimensional surgery – *Morse description*



The local form of a Morse function

for $m = i = 1$

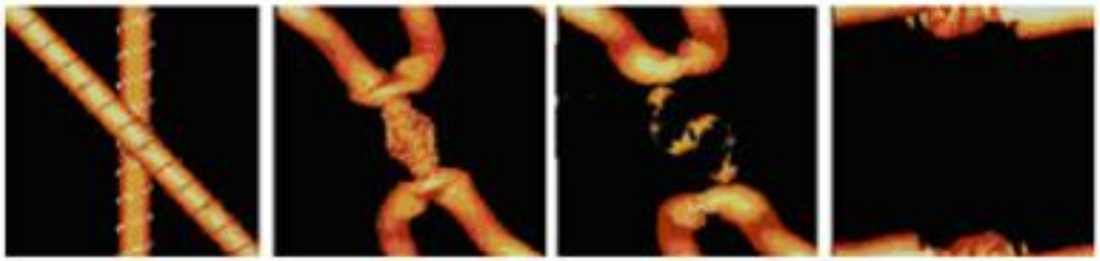
ε -> t



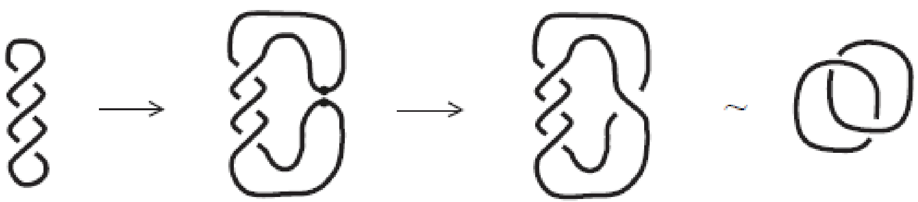
The gradient $\nabla f = (-2x, 2y)$

$\vec{F} = -(\nabla V)$

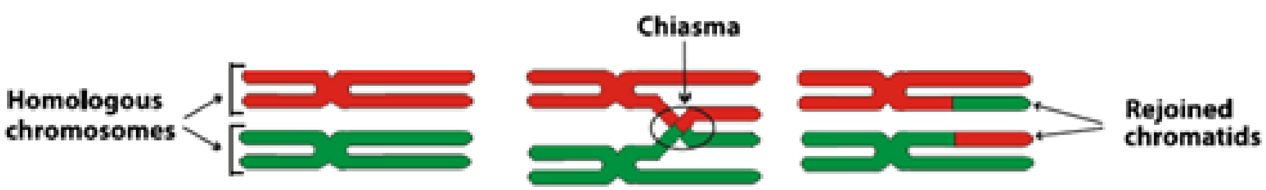
1-dimensional surgery – *Natural processes*



The reconnection of cosmic magnetic lines.



DNA Recombination.



Crossing over of chromosomes during meiosis.

1-dimensional surgery – *Natural processes*

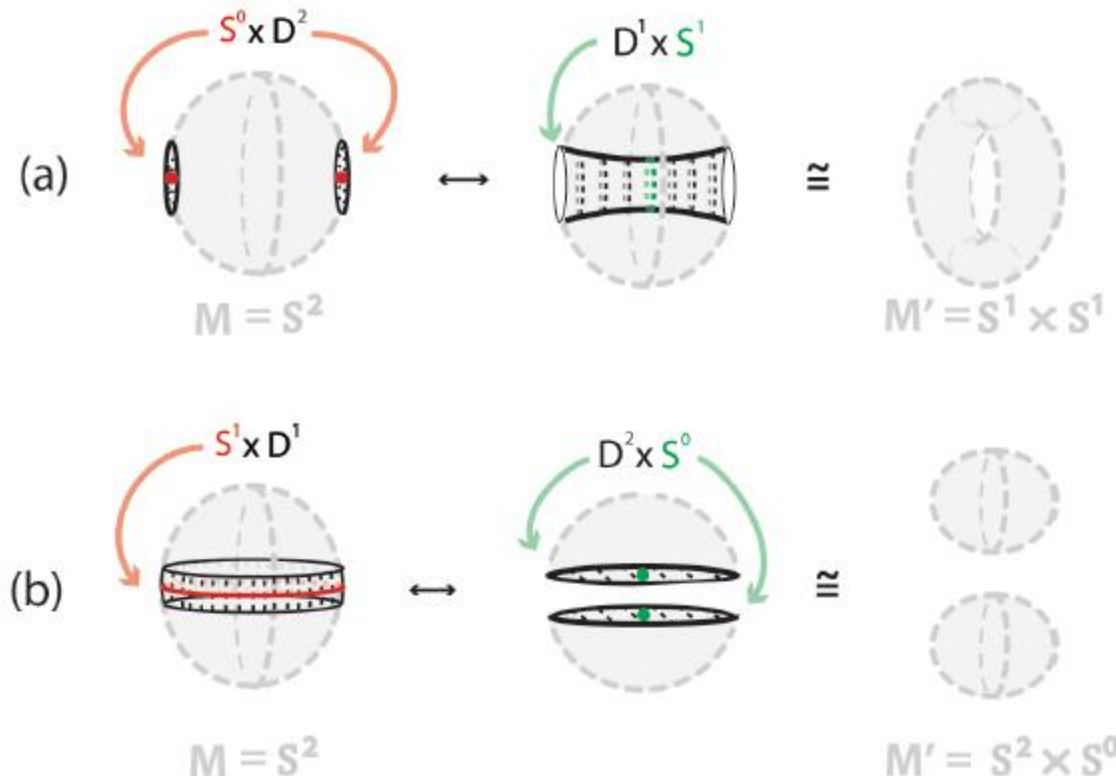
The forces similar to our description in physical phenomena:

-The two **magnetic flux tubes** have a non-zero parallel current through them, which leads to attraction of the tubes.

-During **DNA recombination** enzymes break and rejoin the DNA strands, hence in this case the seeming attraction of the two specified points is realized by the enzyme.

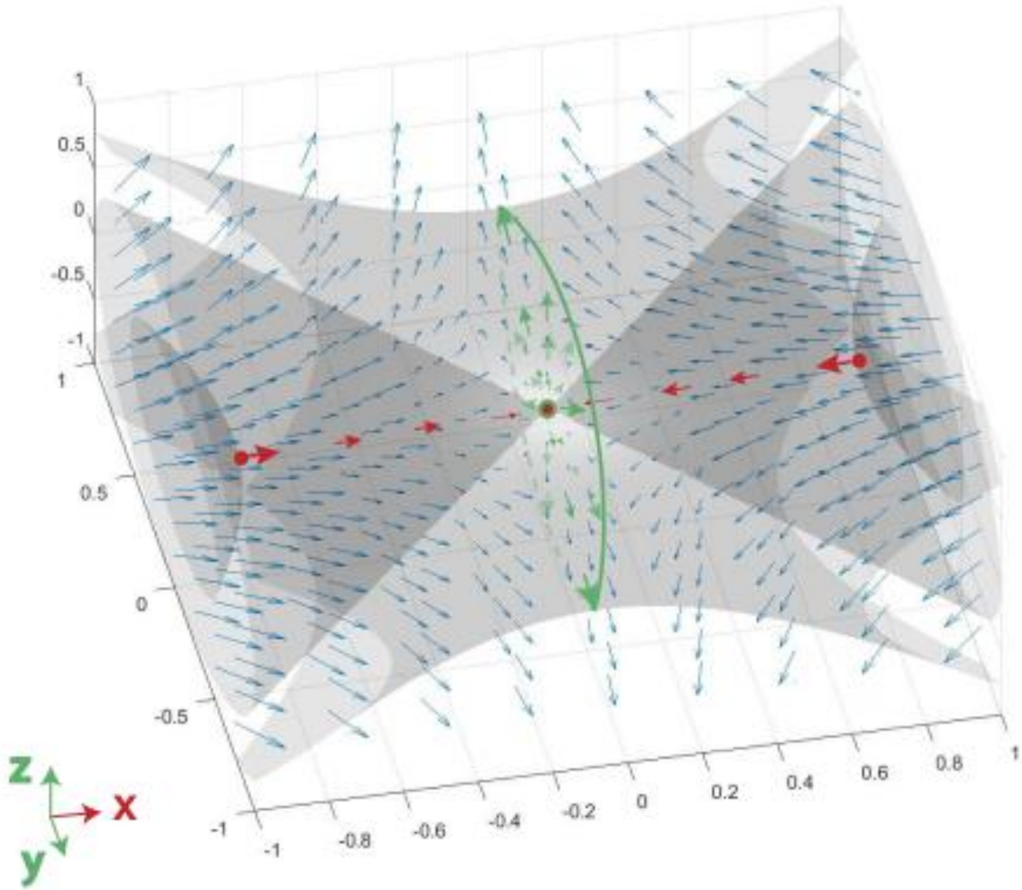
-During **chromosomal crossover** the pairing of the chromosomes is remarkably precise and is caused by mutual attraction of the parts of the chromosomes that are similar or homologous.

2-dimensional surgery – *Definition*



(a) 2-dimensional 0-surgery on $M = S^2$ (b) 2-dimensional 1-surgery on $M = S^2$.

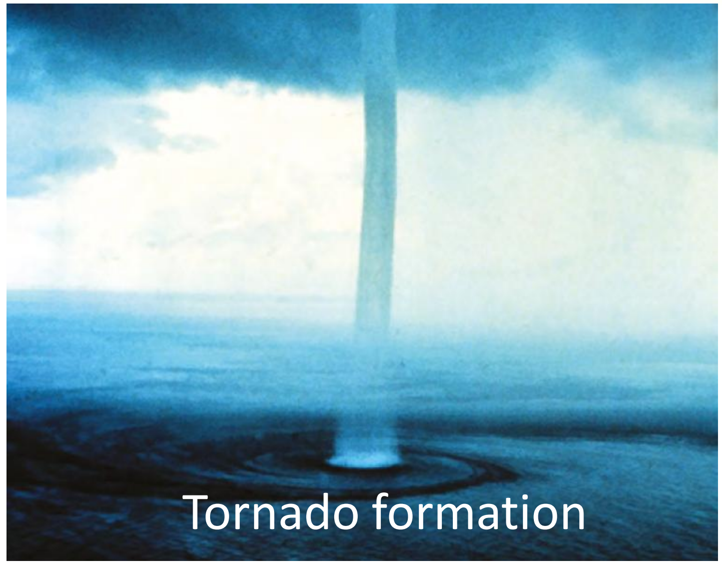
2-dimensional surgery – *Morse description*



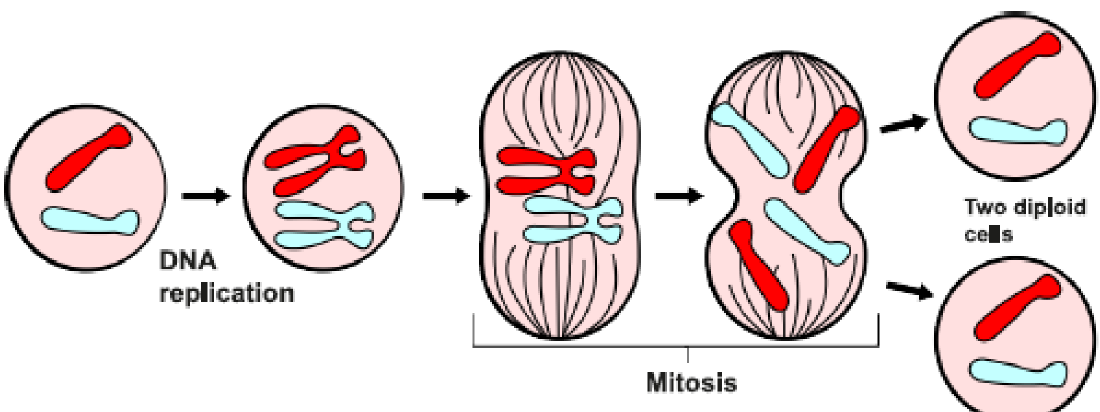
$$g : D^3 \to \mathbb{R}; \quad (x, y, z) \mapsto -x^2 + y^2 + z^2$$

The gradient $\nabla g = (-2x, 2y, 2z)$

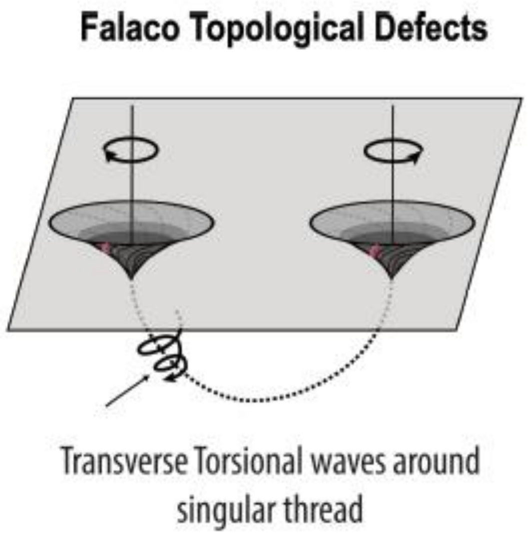
2-dimensional surgery – *Natural processes*



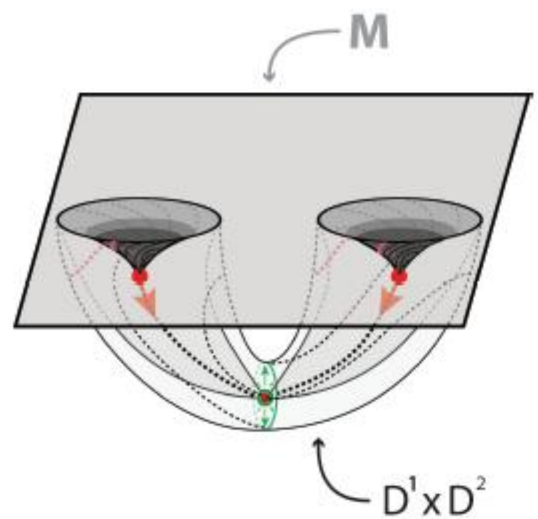
Tornado formation



2-dimensional surgery – *Natural processes*



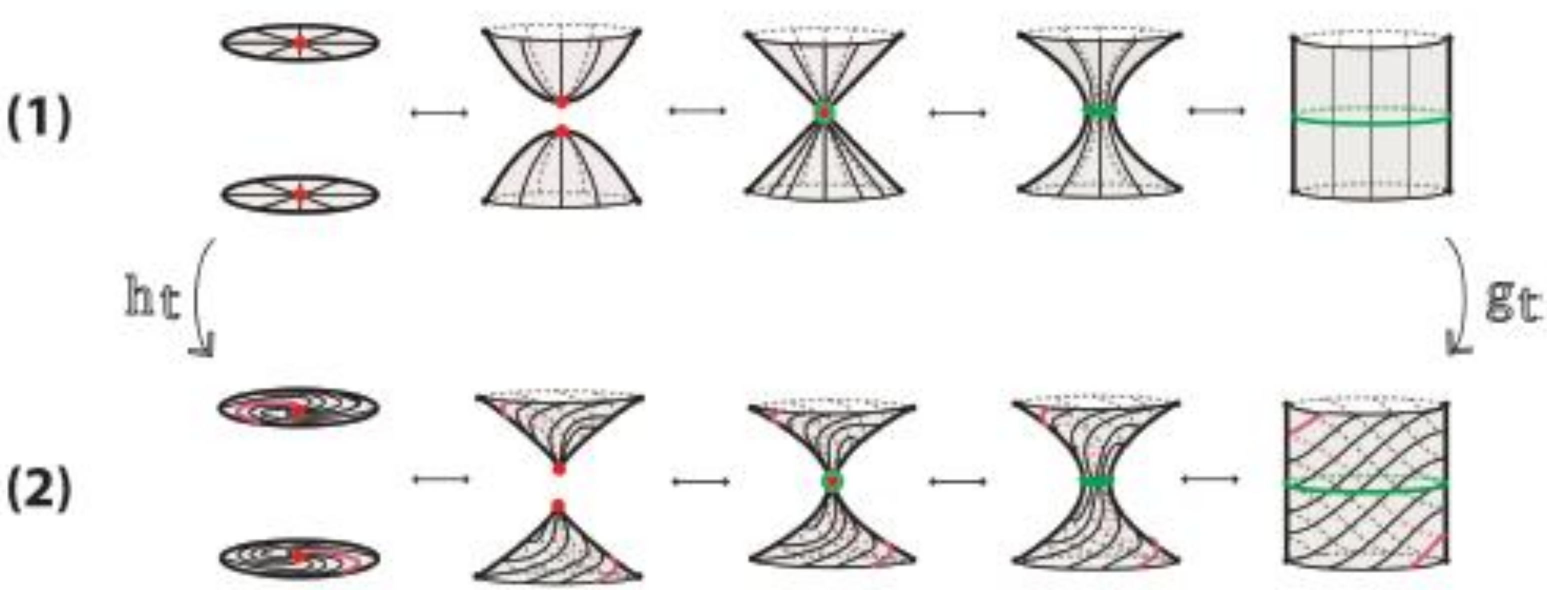
(1)



(2)

(1) Falaco Solitons (2) Homeomorphic representation of handle $D^1 \times D^2$

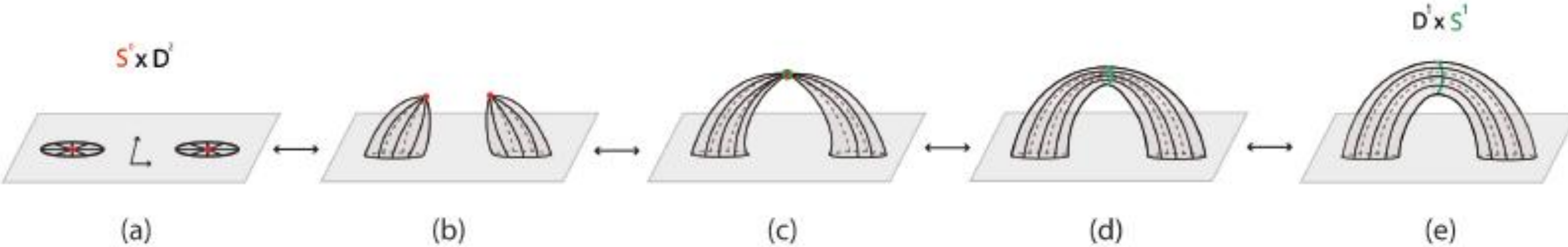
2-dimensional surgery – *Non-trivial embeddings*



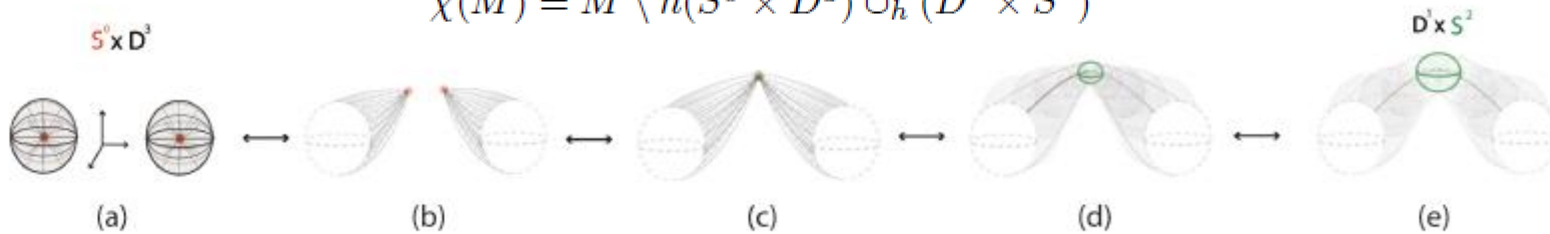
2-dimensional surgery with (1) the standard embedding $h = h_s$ (2) the non-trivial embedding $h = h_t$

If we define the homeomorphisms $\omega_1, \omega_2 : D^2 \to D^2$ to be rotations by $-3\pi/4$ and $3\pi/4$ respectively, then h_t is defined as the composition $h_t : S^0 \times D^2 \xrightarrow{\omega_1 \amalg \omega_2} S^0 \times D^2 \xrightarrow{h} M$.
This rotation induces the twisting g_t of angle $-3\pi/2$ of the final cylinder.

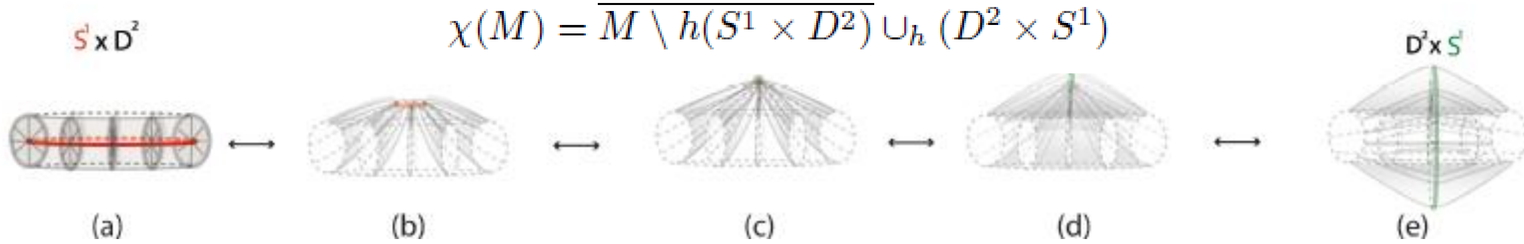
3-dimensional surgery – *Types*

(1)


$$\chi(M) = \overline{M \setminus h(S^0 \times D^3)} \cup_h (D^1 \times S^2)$$

(2)


$$\chi(M) = \overline{M \setminus h(S^1 \times D^2)} \cup_h (D^2 \times S^1)$$

(3)


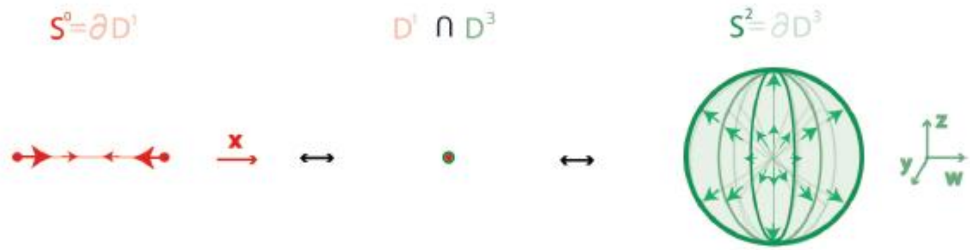
(1) 2-dimensional 0-surgery (2) Outline of 3-dimensional 0-surgery (3) Outline of 3-dimensional 1-surgery

3-dimensional surgery – *Core description*

$$f : D^4 \to \mathbb{R}; \quad (x, y, z, w) \mapsto -x^2 + y^2 + z^2 + w^2$$

$$\nabla f = (-2x, 2y, 2z, 2w)$$

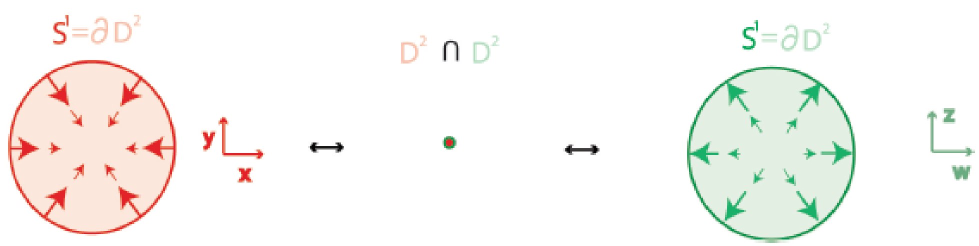
(1)



$$g : D^4 \to \mathbb{R}; \quad (x, y, z, w) \mapsto -x^2 - y^2 + z^2 + w^2$$

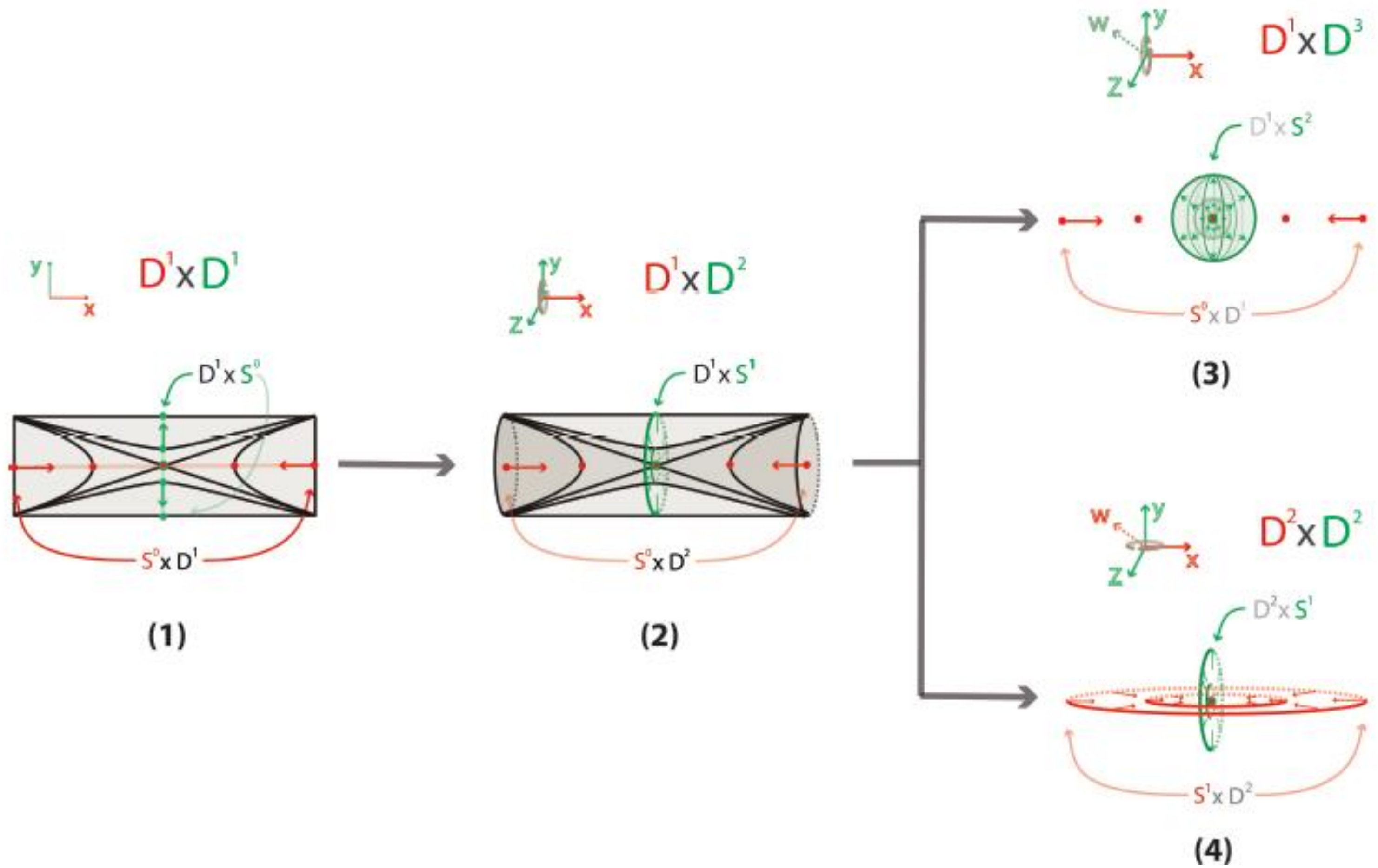
$$\nabla g = (-2x, -2y, 2z, 2w)$$

(2)



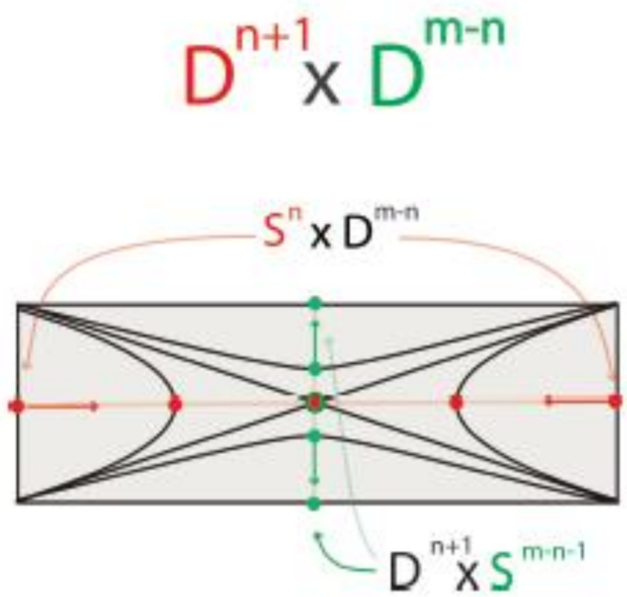
Core view of (1) 3-dimensional 0-surgery (2) 3-dimensional 1-surgery

3-dimensional surgery – *Rotation*



(1) 1-dimensional 0-surgery (2) 2-dimensional 0-surgery via rotation (3) 3-dimensional
(3) 3-dimensional 0-surgery via rotation (4) 3-dimensional 1-surgery via rotation

m-dimensional surgery – *Rotation*

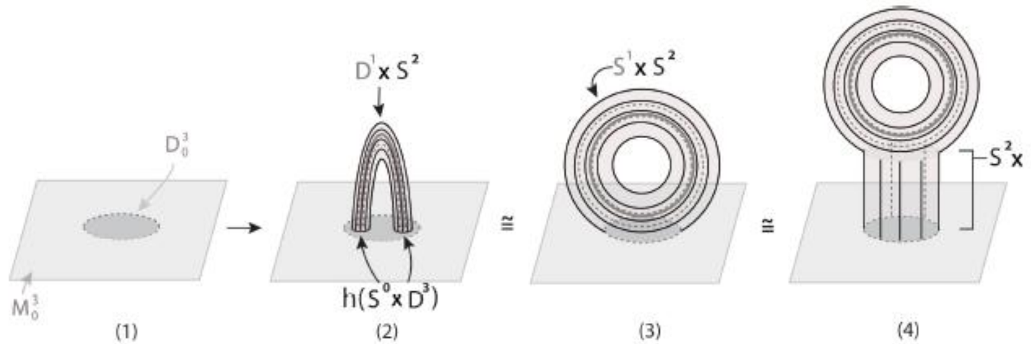


$$-\sum_{j=1}^{n+1} x_j^2 + \sum_{j=n+2}^{m+1} x_j^2 = t, \quad -1 < t < 1$$

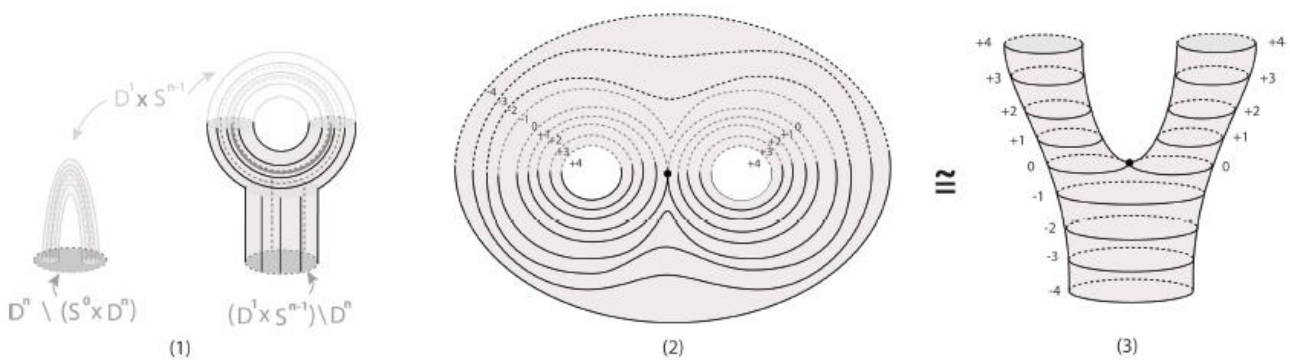
By varying parameter t, one continuously collapses the core S^n of the thickened sphere $S^n \times D^{m-n}$ to the critical point $D^{n+1} \cap D^{m-n}$ from which the core S^{m-n-1} of the thickened sphere $D^{n+1} \times S^{m-n-1}$ uncollapses. The handle $D^{n+1} \times D^{m-n}$ made of these instances can be obtained by $(m-1)$ successive rotations in increasingly higher dimensions of the initial handle $D^1 \times D^1$ made of the instances of 1-dimensional 0-surgery.

3-dimensional 0-surgery – *Topology change*

Proposition . *The result $\chi(M)$ of 3-dimensional 0-surgery on a connected 3-manifold M is homeomorphic to the connected sum $M\#(S^1 \times S^2)$.*



(1) $M = M_0 \cup D_0^3$ (2) $\chi(M)$ (3) $M\#(S^1 \times S^2)$ (4) Homeomorphic representation of $M\#(S^1 \times S^2)$



(1) Removing $(D^1 \times S^{n-1})$ (2), (3) Homeomorphic representations of $D^n \setminus (S^0 \times D^n)$

3-dimensional 0-surgery – *Fundamental group*

The fundamental group of $\chi(M)$ can be characterized using the following lemma which is a consequence of the Seifert–van Kampen theorem:

Lemma . Let $n \geq 3$. Then the fundamental group of a connected sum is the free product of the fundamental group of the components:

$$\pi_1(M\#M') \cong \pi_1(M) * \pi_1(M')$$

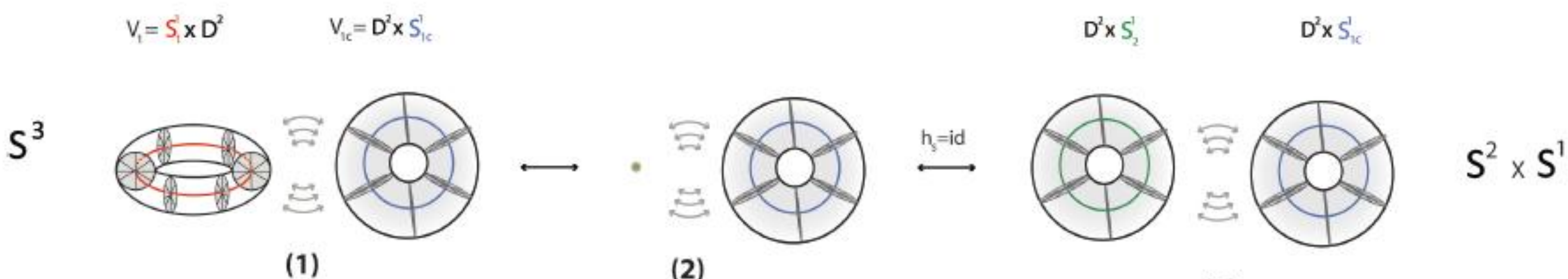
Based on the above, a 3-dimensional 0-surgery on M alter its fundamental group as follows: $\pi_1(\chi(M)) = \pi_1(M\#(S^1 \times S^2)) \cong \pi_1(M) * \pi_1(S^1 \times S^2) \cong \pi_1(M) * (\pi_1(S^1) \times \pi_1(S^2)) \cong \pi_1(M) * \mathbb{Z}$.

3-dimensional 1-surgery – *Topology change & fundamental group*

Theorem . Let K be a blackboard framed knot with longitude $\lambda \in \pi_1(S^3 \setminus N(K))$. Let $\chi_K(S^3)$ denote the 3-manifold obtained by surgery on K with framing longitude λ. Then:

$$\pi_1(\chi_K(S^3)) \cong \frac{\pi_1(S^3 \setminus N(K))}{<\lambda>}$$

where $<\lambda>$ denotes the normal subgroup generated by $\lambda \in \pi_1(S^3 \setminus N(K))$.

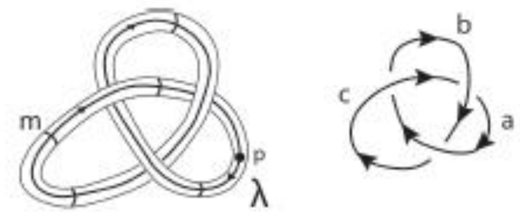


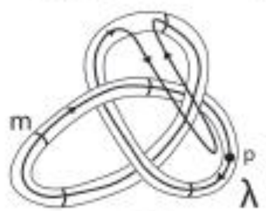
(1) $S^3 = V_1 \cup V_{1c}$ (2) V_1 collapses (3) V_2 emerges

For example, when the identity embedding $h_s(l_1) = m_2$ is used, $\lambda = l_1$ and l_1 is a trivial element in $S^3 \setminus N(K)$ so $<\lambda>=<1>$ The above formula gives us:

$$\pi_1(\chi(S^3)) = \frac{\pi_1(S^3 \setminus h_s(S^1_1 \times D^2))}{<l_1>} = \frac{\pi_1(\mathring{D}^2 \times S^1_{1c})}{<1>} = \frac{\mathbb{Z}}{1} = \mathbb{Z}$$

3-dimensional 1-surgery – *Fundamental group*





$\lambda = cl' = \ldots = cab$

$$\pi_1(T) = (a, b, c | a = b^{-1}cb, b = c^{-1}ac, c = a^{-1}ba).$$

$$\pi_1(\chi(S^3)) = \frac{\pi_1(S^3 \setminus N(T))}{<\lambda>} = \frac{\pi_1(T)}{<\lambda>} = (a, b, c | aba = bab, \lambda = 1)$$

Hence, the fundamental group of the resulting manifold is isomorphic to the binary tetrahedral group $(A, B, C | A^3 = B^3 = C^2 = ABC)$.

Cosmic phenomena - *Topological processes*

- Initial 3-manifold is `space' = 3-dimensional spacial section of 4-dimensional spacetime *(given some natural definition of time, one can slice spacetime up to hypersurfaces which each can be thought of as space)*

- 4-dimensional cobordism bounded by *M* and *x(M)* is *the* 4-dimensional spacetime manifold with past boundary one spacelike component *M* and future boundary the spacelike component *x(M)*

→ A **wormhole** is a tunnel is 3-space connect two spacelike regions, hence we can described it as a *3-dimensional 0-surgery*

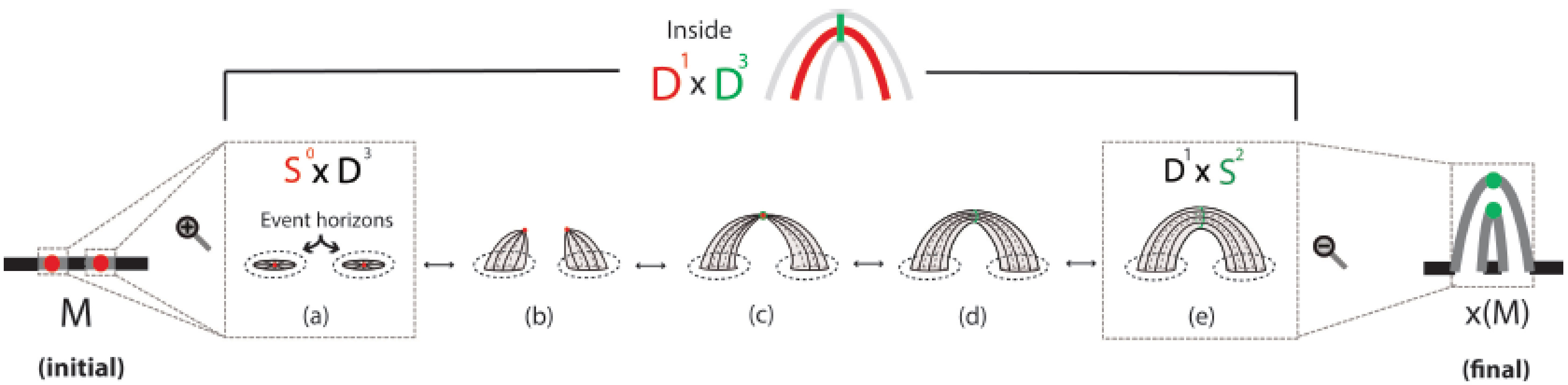
Morse description
Fundamental group

→ A **cosmic string black hole** is a solid torus collapsing is 3-space, hence we can described it as a *3-dimensional 1-surgery*

Wormholes – *Continuous formation*

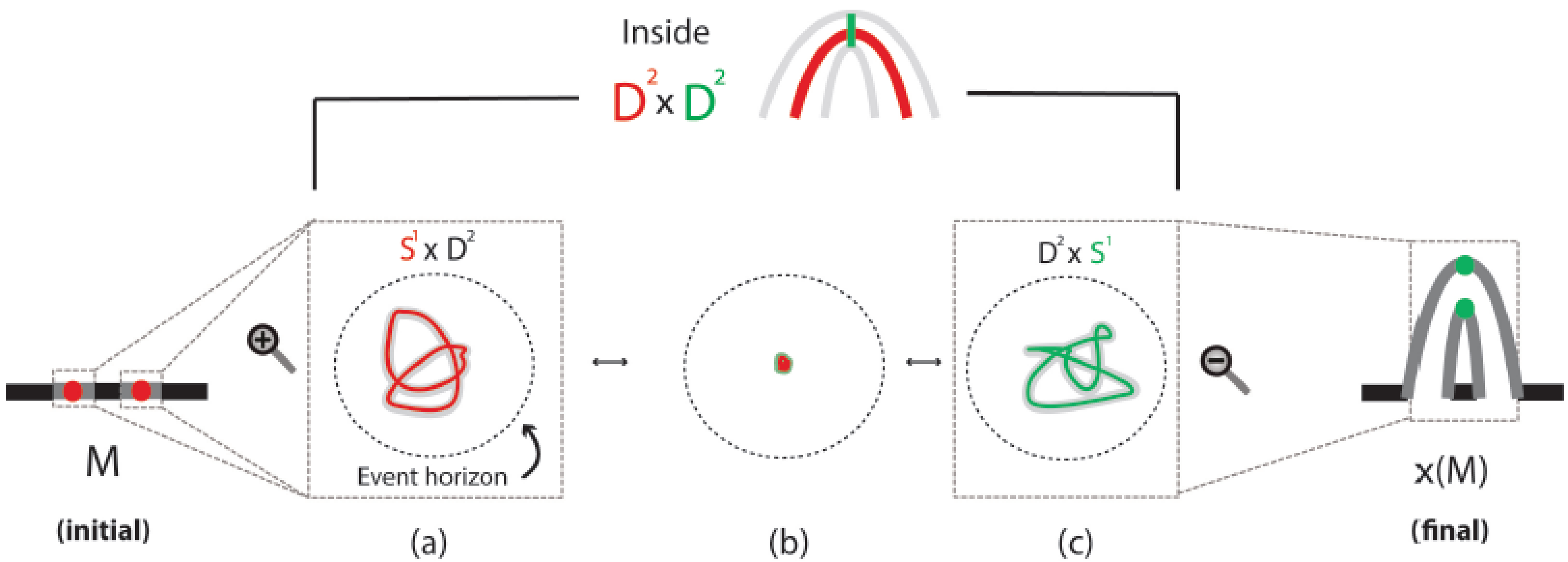
ER= EPR

Wormhole formed by entangled black holes as a continuous change of 3-space
Surgery description proposes a classical path



Entangled black holes connected by a wormhole

Cosmic string black holes - *Avoiding singular manifolds*



3-dimensional 1-surgery inside the event horizon

→ - Physicists undecided whether the prediction of this singularity means that it actually exists or that current knowledge is insufficient.

-The loop shrinks to the critical point which coincides with the physical singularity ***but*** the process continues. We end up with a topologically new universe with a local topology change from the 3-space *M* to the 3-space *x(M).*

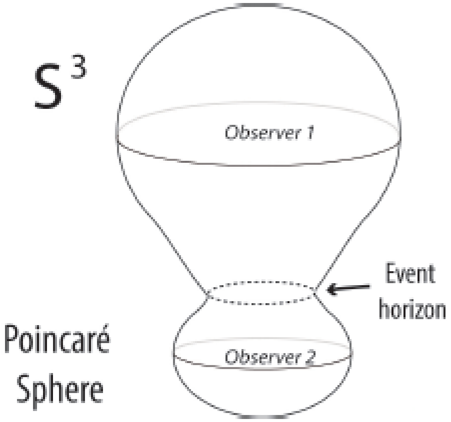
Cosmic string black holes - *New 3-manifolds behind the event horizon*

Theorem (A.H.Wallace and W.Lickorish)

Every closed, connected, orientable 3-manifold can be obtained by surgery on a knot or a link in S^3.

→ • Except from avoiding a singular 3-space, our approach is also consistent with a very large family of 3-manifolds.

• For the Poincaré dodecahedral space (*J.P.Luminet*), the surgery approach suggest that the shape of the universe came about via a knot surgery on the trefoil knot (with the right framing).

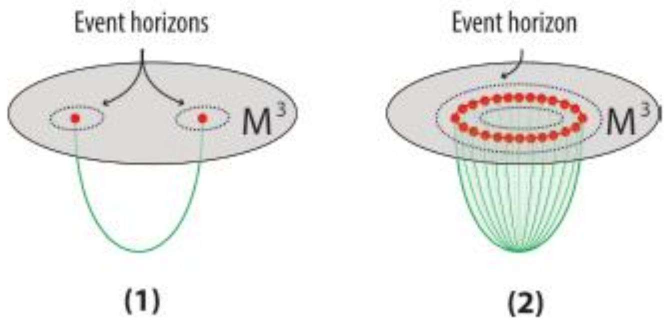


→ We can speculate that finding the Poincaré dodecahedral space *(or some other non-trivial 3-manifold)* in our universe may indicate that we are observers that evolved inside the event horizon of a collapsed trefoil cosmic string *(or some other cosmic string).*

Cosmic string black holes - *Generalized wormholes*

ER= EPR

→ Cosmic string black holes equivalent to wormholes made from a string of entangled black holes.

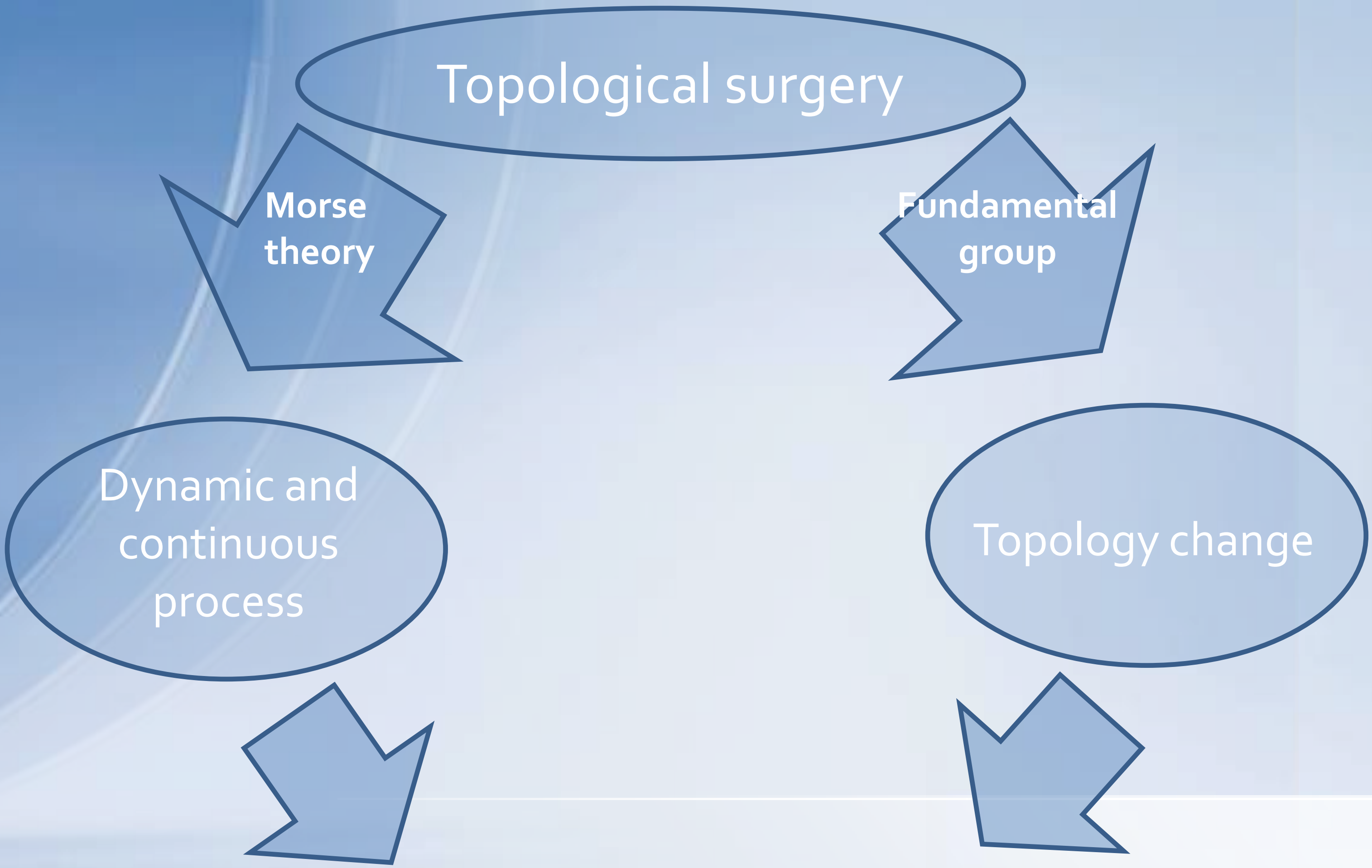


(1) Pair of entangled black holes (2) String of entangled black holes

→ • A cosmic string made of pairs of entangled concentrated masses. Given that all these pairs of masses have started on the same cosmic string, the distinct wormholes merge and the entire collection of wormhole cores (the green arcs, see (1)) forms a 2-disc (see (2)).

• A cosmic string black hole can be seen as a collection of Einstein-Rosen bridges. The process of surgery amalgamates these bridges to form a new 3-manifold resulting from surgery on the cosmic string. From any black hole location on the cosmic string to any other, there is a `bridge' through the new 3-manifold.

Conclusion



Description of surgery and related natural process in all 3 dimensions

ER=EPR, connectivity of space and topology

Physical implications of the surgery approach (in all phenomena of dim 1,2,3)

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Thank you