Laboratory of Topology and Dynamics,
Novosibirsk State University

and

Center Poncelet

Conference booklet of multidisciplinary workshop

"Topological structures in mathematics, physics and biology"*

14-18 September 2018, Novosibirsk

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Conference Schedule

	15 september	16 september	17 september
10:30 - 11:20	Louis Kauffman Virtual Knot Cobordism	Sergei Matveev Newman's Diamond Lemma and prime decompositions of knots in thickened surfaces	Yuri Kordyukov Asymptotic spectral analysis of Toeplitz operators on symplectic manifolds
11:20 - 11:40		Coffee break	
11:40 - 12:30	Roman Novikov Non-abelian Radon transform and its applications	Stathis Antoniou Topological surgery in cosmic phenomena	Joanna Sulkowska From a single protein to a pair of chromosomes - Mysteries of entanglement
12:30 - 12:40		Small break	
12:40 - 13:30	Peter Virnau Knots in DNA, Chromosomes and Polymer Melts	Andrei Shafarevich Semiclassical quantization of complex vector bundles over singular invariant curves of Hamiltonian systems	Ivan Dynnikov Rectangular diagrams of surfaces and mirror diagrams
13:30 - 15:00		Lunch	
15:00 - 15:50	Valeriy Bardakov Pseudo knots and pseudo braids	Rukhsan Haq Topological Protection of Majorana Fermion Qubits	Pedro Lopes The hyperfinite algorithm for "sequences" of knots
15:50 - 16:10		Coffee break	
16:10 - 17:00	Sergei Nechaev Number-theoretic aspects of 1D localization: from spectral statistics of sparse random graphs to the Dedekind eta-function	Ljubica Velimirovic On small deformations of knots	

TOPOLOGICAL SURGERY IN COSMIC PHENOMENA

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Topological surgery is a mathematical technique used for creating new manifolds out of known ones. We observe that it occurs in natural phenomena where forces are applied and the manifold in which they occur changes type. Using Morse theory, we provide a way to formalize the temporal evolution of these processes. We apply this description to various phenomena exhibiting 1 and 2-dimensional surgery such as DNA recombination and the formation of Falaco solitons. We then propose the temporal evolution of 3-dimensional surgery as a new description of the formation of wormholes and black holes. Finally, we analyze the cosmological implications of this topological approach. We hope that through this study, topology and dynamics of many natural phenomena, as well as topological surgery itself, will be better understood.

PSEUDO KNOTS AND PSEUDO BRAIDS

VALERIY BARDAKOV

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Definition The *monoid of pseudo braids* PM_n is a monoid generated by σ_i, σ_i^{-1}, p_i, $i = 1, 2, \ldots, n-1$, where the elements $\sigma_i^{\pm 1}$ generate the braid group B_n and generators p_i satisfy the defining relations

$$\begin{aligned}
p_i p_j &= p_j p_i, \qquad |i-j| \geq 2,\\
p_i \sigma_j^{\pm 1} &= \sigma_j^{\pm 1} p_i, \qquad |i-j| \geq 2,\\
p_i \sigma_i^{\pm 1} &= \sigma_i^{\pm 1} p_i, \qquad i = 1, 2, \ldots, n-1,\\
\sigma_{i+1}\sigma_i p_{i+1} &= p_i \sigma_{i+1}\sigma_i, \qquad i = 1, 2, \ldots, n-2,\\
\sigma_i \sigma_{i+1} p_i &= p_{i+1}\sigma_i\sigma_{i+1}, \qquad i = 1, 2, \ldots, n-2.
\end{aligned}$$

The *group of pseudo braids* PB_n is a group generated by σ_i, p_i, $i = 1, 2, \ldots, n-1$ and defined by the same defining relations as PM_n.

Proposition. *The monoid of pseudo braids PM_n is isomorphic to the singular braid monoid SM_n and the group of pseudo braids PB_n is isomorphic to the group of singular braids SB_n for all $n \geq 2$.*

Theorem. *Let L and L' be two pseudo link diagrams. Suppose that $L = \widehat{\beta}$ and $L' = \widehat{\beta'}$ for some pseudo braids $\beta \in PM_n$ and $\beta' \in PM_m$. Then the pseudo links L and L' are equivalent if and only if there is a finite sequence of Markov's moves (M1-M4), which transform β to β', where*

M1. If $\beta \in PM_n$ and $a \in B_n$ then

$$\beta \leftrightarrow a^{-1}\beta a.$$

M2. If $\beta = \beta_1\beta_2 \in PM_n$ then

$$\beta \leftrightarrow \beta_2\beta_1.$$

M3. If $\beta \in PM_n$ then

$$\beta \leftrightarrow \beta\sigma_n^{\pm 1} \in PM_{n+1}.$$

M4. If $\beta \in PM_n$ then

$$\beta \leftrightarrow \beta p_n \in PM_{n+1}.$$

This is joint work with S. Jablan, H. Wang. The main results are published in [2].

References

[1] A. Henrich, R. Hoberg, S. Jablan, L. Johnson, E. Minten, L. Radovic, *The Theory of Pseudoknots*, J. Knot Theory Ramifications, 22, No. 7 (2013), 21 pp.

[2] V. G. Bardakov, S. Jablan, H. Wang, *Monoid and group of pseudo braids*, Journal of Knot Theory and Its Ramifications, 25, No. 9 (2016) 13 pages. DOI: 10.1142/S0218216516410029.

RECTANGULAR DIAGRAMS OF SURFACES AND MIRROR DIAGRAMS

IVAN DYNNIKOV

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I will describe the machinery that we worked out with Maxim Prasolov for studying rectangular diagrams of knots. Our formalism provides a convinient combinatorial presentation of knotted surfaces in the three-space and of spatial ribbon graphs. There are simple rules to define the diagrams and simple 'Reidemester moves'. The formalism also appears to be perfectly suitable as a combinatorial model for Giroux's convex surfaces.

TOPOLOGICAL PROTECTION OF MAJORANA FERMION QUBITS

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Majorana fermions have recently attracted a lot of attention due to their promising role in topological quantum computation. Majorana fermions exhibit anyonic statistics and hence give rise to braid group representations which makes them ideally suited to braiding operations in quantum computer. In my presentation I will talk about braid group representation of Majorana fermions and show that how these representations are different from the ones based on spin operators. Another aspect of Majorana fermions which makes them important for quantum computation is that they can store information non-locally and hence it is inherently robust to local perturbations. We will show that Majorana fermions exist as edge modes in systems with Z_2 topological order. In such systems there are emergent symmetries which lead to topological protection. We will present a theoretical formulation for the topological protection of Majorana fermion qubits and show that as long as there exist these emergent symmetries in the system, qubits and hence the quantum information is topologically protected.

VIRTUAL KNOT COBORDISM

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This talk will discuss cobordism of knots and virtual knots. Classical knot cobordism can be defined combinatorially by adding extra moves to the Reidemeister moves corresponding to the birth of an unknotted component, the death of an unknotted component and the movement through a saddle point. We generalize this combinatorial description of knot and link cobordism to virtual knots and links. Combinatorially, virtual knots and links are represented by diagrams with the usual crossings and with virtual crossings corresponding to the immersion of the abstract link diagram into the plane. Generalized Reidemeister moves for virtuals include the classical Reidemeister moves plus detour moves for the virtual crossings. By adding deaths, births and saddles we obtain a theory of virtual link cobordism. Many classical questions and structures generalize. Virtual knots and links bound virtual surfaces and one can ask for the least genus of such a surface. This is called the four ball genus $g_4(K)$ for a virtual knot of link K. We describe how we use Khovanov homology for virtual knots to determine the g_4 of positive virtual links, and we describe our recent results about the concordance invariance of the affine index polynomial invariant of virtual knots and links.

ASYMPTOTIC SPECTRAL ANALYSIS OF TOEPLITZ OPERATORS ON SYMPLECTIC MANIFOLDS

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We describe the algebra of Toeplitz operators on a quantizable compact symplectic manifold associated with the renormalized Bochner Laplacian of a prequantum line bundle. This algebra provides a Berezin-Toeplitz type quantization of the symplectic manifold. It can also be considered as a generalization of the algebra of semiclassical pseudodifferential operators. We discuss asymptotic spectral properties of Toeplitz operators in the semiclassical limit such as the asymptotic behavior of low-lying eigenvalues and localization properties of the corresponding eigenfunctions, as well as some applications to the spectral theory of the Bochner Laplacian.

THE HYPERFINITE ALGORITHM FOR "SEQUENCES" OF KNOTS

PEDRO LOPES

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We consider sequences of knots and whether these sequences converge or not. More precisely, we consider sequences of equivalence classes of knots corresponding to appropriately chosen invariants taking values in complete metric spaces - the hyperfinite algorithm. These invariants stem from the CJKLS invariants for knots to which we apply a "thermodynamic limit". (Inspiration from Statistical Mechanics of Exactly Solved Models should be noted here.) Examples of such convergent "sequences" and their limits (hyperfinite knots) are given. The stability of these notions with respect to the change of invariant is addressed.

NEWMAN'S DIAMOND LEMMA AND PRIME DECOMPOSITIONS OF KNOTS IN THICKENED SURFACES

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We prove that any homologically trivial knot in a thickened surface admits a prime decomposition into connected sum of prime knots. Moreover, the summands of the decomposition are unique up to equivalence. For homologically essential knots this may be wrong.

NUMBER-THEORETIC ASPECTS OF 1D LOCALIZATION: FROM SPECTRAL STATISTICS OF SPARSE RANDOM GRAPHS TO THE DEDEKIND ETA-FUNCTION

SERGEI NECHAEV

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We discuss the number-theoretic properties of distributions appearing in physical systems when an observable is a quotient of two independent exponentially weighted integers. The spectral density of ensemble of linear chains (graphs) weighted exponentially f^L ($0 < f < 1$), where L is the chain length, serves as a particular example. At $f \to 1$, the spectral density can be expressed through the discontinuous at all rational points, Thomae ("raindrop") function. We suggest a continuous approximation of the raindrop function, based on the Dedekind eta-function near the real axis. We provide simple arguments, based on the "Euclid orchard" construction, demonstrating the presence of Lifshitz tails, typical for 1D Anderson localization, at spectral edges. We also pay attention to the connection of the Dedekind eta-function near the real axis to phyllotaxis and invariant measures of some continued fractions.

NON-ABELIAN RADON TRANSFORM AND ITS APPLICATIONS

ROMAN NOVIKOV

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Considerations of the non-abelian Radon transform were started in [Manakov, Zakharov, 1981] in the framework of the theory of solitons in dimension 2+1. On the other hand, the problem of inversion of transforms of such a type arises in differential geometry and in different tomographies, including emission tomographies, polarization tomographies, and vector field tomography. In this talk we give a short review of old and recent results on this subject.

References

[1] R.G. Novikov, Non-abelian Radon transform and its applications, https://hal.archives-ouvertes.fr/hal-01772611v1

SEMICLASSICAL QUANTIZATION OF COMPLEX VECTOR BUNDLES OVER SINGULAR INVARIANT CURVES OF HAMILTONIAN SYSTEMS

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Semiclassical asymptotics of eigenvalues of quantum operators can be associated with invariant geomertical objects of corresponding classical Hamiltonian systems. In particular, certain sequences of eigenvalues can correspond to special complex vector bundles over isotropic manifolds (Maslov complex germ). We study eigenvalues, corresponding to singular sets - namely, to vector bundles over singular invariant curves of partially integrable Hamiltonian systems.

FROM A SINGLE PROTEIN TO A PAIR OF CHROMOSOMES - MYSTERIES OF ENTANGLEMENT

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Topology plays an ever-increasing role in modern life sciences since the discovery and artificial creation of knots in DNA and proteins. Nowadays, at least 6% of proteins are known to be knotted, slipknotted, linked [1] or to contain lassos [2], even though for a long time it was believed that proteins should not be entangled. Knotted proteins are believed to be functionally advantageous and to provide extra stability to protein chains. During the talk, I will discuss available methods to analyze entanglement in the open chains and present the new type of entanglement in proteins.
It is well-known that the 3D structure of the genome plays a critical role in regulating gene expression. Recent developments have for the first time enabled the determination of three-dimensional structures of individual chromosomes and genomes based on Hi-C chromosome conformation contact data. While some model structures are highly knotted, other models based on Hi-C data predict only few knots. Moreover recently we found that maybe some chromosomes could be linked. Even though the abundance of entanglements in chromosomes is still controversial, there is a clear need to check model structures for entanglements, in particular when higher resolution data becomes available in the near future. During the talk, I will present the KnotGenome server the first server that detects and characterizes knots in single chromosomes, as well as links between chromosomes [2].

References

[1] Dabrowski-Tumanski, Sulkowska, Topological knots and links in proteins, PNAS (2017) 114, 34153420
[2] Niemyska et al., Complex lasso: new entangled motifs in proteins, Scientific Reports (2016) doi:10.1038/srep36895
[3] Sulkowska et al., KnotGenom: server to detect entanglement in and between chromosomes, NAR (2018)

ON SMALL DEFORMATIONS OF KNOTS

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Talk will tell the story of knots and change of shape descriptors of knots during infinitesimal bending.

Problem of infinitesimal bending of knots is a special part of theory of deformation. Infinitesimal bending is not an isometric deformation, or roughly speaking it is with appropriate precision. Arc length is stationary under infinitesimal bending. The physical properties of knotted and linked configurations in space have long been of interest to mathematicians. More recently, these properties have become significant to biologists, physicists, and engineers among others. Their depth of importance and breadth of application are now widely appreciated and valuable progress continues to be made each year.

Physical knot theory is the study of mathematical models of knotting phenomena, often motivated by considerations from biology, chemistry, and physics (Kauffman 1991). Physical knot theory is used to study how geometric and topological characteristics of filamentary structures, such as magnetic flux tubes, vortex filaments, polymers, DNA's, influence their physical properties and functions. In this talk we consider infinitesimal bending of the first and the second order of curves and knots. The total curvature of the knot during the second order infinitesimal bending is discussed and the first and the second variation of the total curvature are given. Some examples aimed to illustrate infinitesimal bending of knots are given.

Values of curvature at points of knots are scaled and appropriate colours used to illustrate curvature.

Total curvature of bent knots are numerically calculated and illustrated.

KNOTS IN DNA, CHROMOSOMES AND POLYMER MELTS

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In this talk I will mainly focus on three of our more recent projects: First, I will present results on using knots as a gauge for the development of coarse-grained models for DNA and polymers. We show that single DNA chains exceeding 250,000 base pairs in physiologically relevant salt conditions tend to be knotted in agreement with recent experiments. The analysis is motivated by the emergence of DNA nanopore sequencing technology, as knots are a potential cause of erroneous nucleotide reads in nanopore sequencing devices and may severely limit read lengths in the foreseeable future. Recent developments have for the first time allowed the determination of three-dimensional structures of individual chromosomes and genomes in nuclei of single haploid mouse embryonic stem (ES) cells based on Hi-C chromosome conformation contact data. In this project, we further analyze these structures and provide the first evidence that G1 phase chromosomes are knotted, consistent with the fact that plots of contact probability vs sequence separation show a power law dependence that is intermediate between that of a fractal globule and an equilibrium structure. Finally, we investigate the occurrence of knots in polymer melts. In polymer physics it is typically assumed that chains in melts can be described by effective random walks without excluded volume interactions. We show that this idea is problematic as the latter severely overrate the occurrence of knots. Interestingly, we find that the structure of a chain in a melt is very similar to the structure of a single chain undergoing a collapse transition at the Theta-point, which in turn are not well-represented by random walks either.

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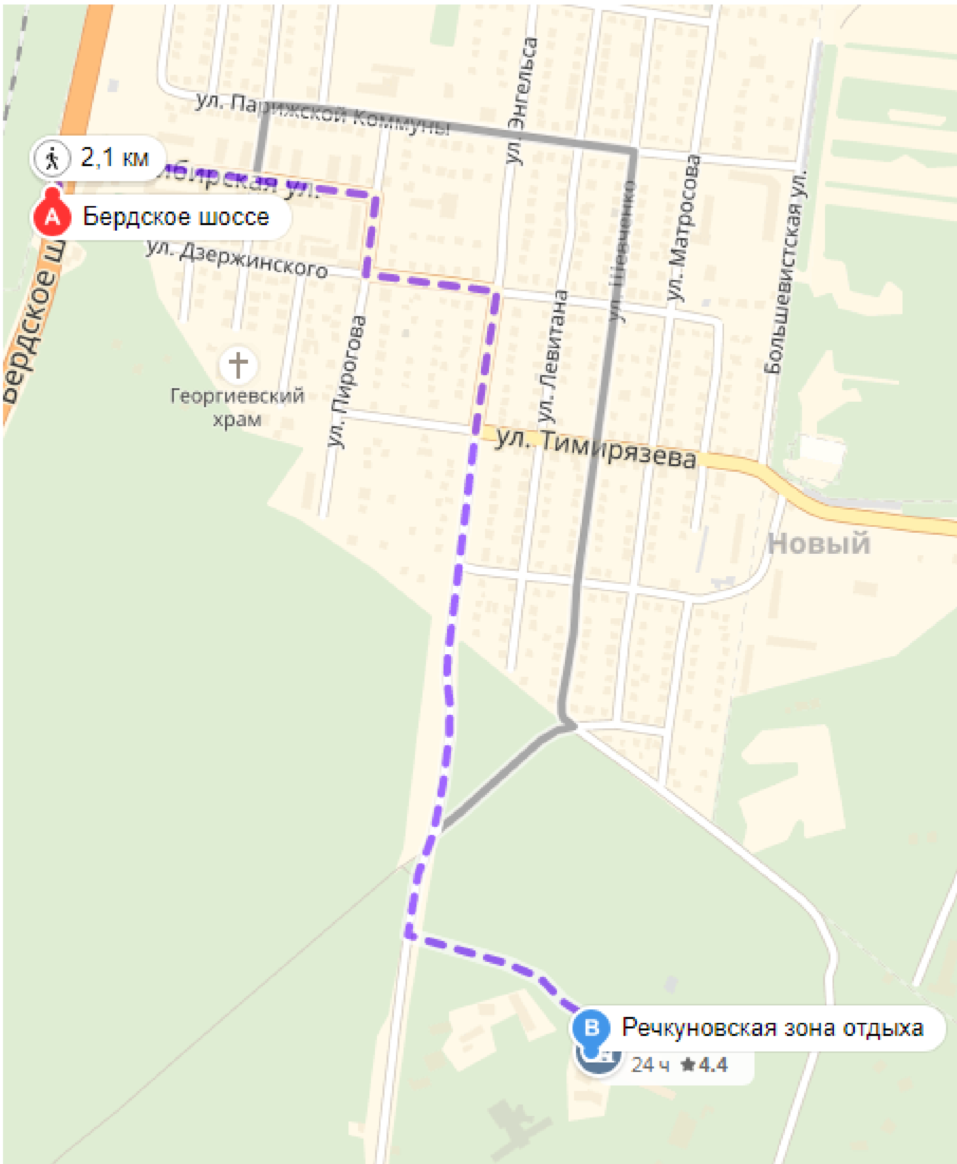
The conference will be held in the picturesque surroundings of Novosibirsk in resort hotel Sosnovka.

Transport.

Taking a taxi is the easiest way to arrive Sosnovka.
For the public transportation, please, use the bus stop Novyy poselok, there is scheme of the way from bus stop to Sosnovka below.



Meal.

The conference will provide breakfast (9:00–10:30), lunch (13:30–15:00) and dinner (18:00–19:30) from 15 to 17 September, dinner at 14 September and breakfast at 18 September.

Also there are restaurants on the territory which work till midnight.

A conference dinner is scheduled from 18 to 22 on September 26 in the restaurant Veselo-Selo on the territory of Sosnovka.

Additional.

We have booked a sauna on the evening of Monday, 17 September. The sauna costs 2500 rubles per hour for a group up to 10 people.

Threre is the schematic map of Sosnovka below. The building marked OB is Office Building, our sessions will be held in this building. The reception is placed on the ground floor of OB. B1, B2, B3 is living buildings.